AMS316 HW5

(Due Nov 30, 2011)

1. Following the procedure I have done in class, show that the ACFs of the $ARMA(1,1)$ model,

$$X_t = \alpha X_{t-1} + Z_t + \beta Z_{t-1}, \qquad |\alpha| < 1, |\beta| < 1,$$

is given by

$$\rho(1) = (1+\alpha\beta)(\alpha+\beta)/\left(1+\beta^2+2\alpha\beta\right)$$
$$\rho(k) = \alpha\rho(k-1), k = 2, 3, ...$$

2. For the model $(1-B)(1-0.2B)X_t = (1-0.5B)Z_t$:

   (a) Classify the model as an $ARIMA(p,d,q)$ process(i.e. find p,d,q).

   (b) Determine whether the process is stationary and invertible.

   (c) Evaluate the first three MA coefficients of the model when expressed as $MA(\infty)$ model.

   (d) Evaluate the first three AR coefficients of the model when expressed as $AR(\infty)$ model.

3. Show that AR(2) process

$$X_t = X_{t-1} + cX_{t-2} + Z_t$$

is stationary provided $-1 < c < 0$. Find the autocorrelation function when $c = -3/16$.

4. Show that the $AR(3)$process

$$X_t = X_{t-1} + cX_{t-2} - cX_{t-3} + Z_t$$

is non-stationary for all values of c.