AMS316 HW6

(Due Dec 12, 2011)

1. Consider the AR(1) process, $X_t = \mu + \alpha X_{t-1} + Z_t$, where Z_t are i.i.d. standard normal random variables. Derive the least square estimates for μ and α by minimizing

$$S(\mu, \alpha) = \sum_{t=1}^{n} \left(X_t - \mu - \alpha X_{t-1} \right)^2.$$

2. For the MA(1) model given by $X_t = Z_t + \theta Z_{t-1}$ and observations $X_1, \ldots, X_N$, show that the 1-step ahead forecast $\widehat{X}_N(1) = \theta Z_N$ and that the h-step ahead forecast $\widehat{X}_N(h) = 0$ for $h = 2, 3, \ldots$.

3. For the AR(1) model given by $X_t = \alpha X_{t-1} + Z_t$ and observations $X_1, \ldots, X_N$, show that $\widehat{X}_N(h) = \alpha^h X_N$ for $h = 1, 2, \ldots$. For the AR(1) model with non-zero mean μ, given by $Y_t - \mu = \alpha(Y_{t-1} - \mu) + Z_t$ and observations $Y_1, \ldots, Y_N$, show that $\widehat{Y}_N(h) = \mu + \alpha^h (Y_N - \mu)$ for $h = 1, 2, \ldots$.