AMS316 Sample Midterm

This is the AMS316 miderm in fall 2010.

1. Suppose we have a seasonal seris of monthly observations $\{X_t\}$, for which the seasonal factor at time t is denoted by $\{S_t\}$. We further assume that the seasonal pattern is constant through time so that $S_t = S_{t-12}$ for all t. Denote a stationary seris of random variables by $\{\epsilon_t\}$. Consider the model $X_t = bt + S_t + \epsilon_t$ having a global linear trend and additive seasonality. Show that the seasonal difference operator ∇_{12} acts on X_t to produce a stationary series. (Hints: Please use only heuristic arguments about stationarity as you did in your HW.)

2. Consider the series $X_t = a + bt + ct^2$, where a, b and c are constants. Compute its first- and second-order differences ∇X_t and $\nabla^2 X_t$.

3. Given N observations $x_1, \ldots x_N$, on a time series. (a) Write down the estimation formula for **autocorrelation** coefficient at lag 1. (b) In general, what is your estimate for autocorrelation coefficient at lag k.

4. Suppose that a stationary stochastic process $X(t)$ have autocovariance function (acvf) $\gamma_X(\tau)$ and autocorrelation function (acf) $\rho_X(\tau)$. Show by the definition of acvf and acf that, $\gamma_X(\tau) = \gamma_X(-\tau)$ for $\tau = 1, 2, \ldots$ and $\rho_X(0) = 1$.

5. Write down the definition of the second-order (or weakly) stationary process.

6. Consider the purely random processes $\{Z_t\}$, where Z_t are independent and identically distributed random variables with mean 0 and variance σ_Z^2. (a) Compute its autocovariance function $\gamma_Z(\tau)$ and autocorrelation function $\rho_Z(\tau)$ for $\tau = 0, 1, 2, \ldots$. (b) Is this process weakly statioanry?

7. Consider the MA(1) process: $X_t = Z_t + \theta Z_{t-1}$ where $|\theta| < 1$ and Z_t are independent and identically distributed random variables with mean 0 and variance σ_Z^2. (a) Compute its autocovariance function $\gamma_Z(\tau)$ and autocorrelation function $\rho_Z(\tau)$ for $\tau = 0, 1, 2, \ldots$. (b) Is this process weakly statioanry?