Quiz One Solution

1

$$\begin{aligned} \bigtriangledown_{12} X_t &= X_t - X_{t-12} = b_t + S_t + \epsilon_t - b_{t-12} - S_{t-12} - \epsilon_{t-12} \\ &= 12b + \epsilon_t - \epsilon_{t-12} \end{aligned} \tag{1}$$

2

$$\begin{aligned} \bigtriangledown X_t &= X_t - X_{t-1} \\ &= a + bt + ct^2 - \big(a + b(t-1) + c(t-1)^2\big) \\ &= b + 2tc - c \end{aligned} \tag{2}$$

$$\begin{aligned} \bigtriangledown^2 X_t &= \bigtriangledown X_t - \bigtriangledown X_{t-1} \\ &= b + 2tc - c \\ &- \big(a + b(t-1) + c(t-1)^2 - (a + b(t-2) + c(t-2)^2)\big) \\ &= 2c \end{aligned} \tag{3}$$

3

$$\gamma_1 = \frac{\sum_{t=1}^{N-1} (X_t - \overline{X})(X_{t+1} - \overline{X})}{\sum_{t=1}^{N} (X_t - \overline{X})^2} \tag{4}$$

$$\gamma_k = \frac{\sum_{t=1}^{N-k} (X_t - \overline{X})(X_{t+k} - \overline{X})}{\sum_{t=1}^{N} (X_t - \overline{X})^2} \tag{5}$$

Where

$$\overline{X} = \frac{\sum_{t=1}^{N} X_t}{N} \tag{6}$$

4

$$\gamma_x(h) = cov(X_t, X_{t+h}) = cov(X_{t-h}, X_t) = \gamma_x(-h) \tag{7}$$

$$\rho_x(k) = cov(X_t, X_{t+k}) = \frac{\gamma_x(k)}{\gamma_x(0)} \tag{8}$$

$$\rho_x(0) = \frac{\gamma_x(0)}{\gamma_x(0)}) = 1 \tag{9}$$

5 A process is called second-order stationary if its mean is constant and its acv.f. depends only on the lag, so that

$$E(X_t) = \mu \tag{10}$$

$$cov(X_t, X_{t+h}) = \gamma(h) \tag{11}$$

6 a)

$$\gamma_z(h) = cov(Z_t, Z_{t+h}) = \begin{cases} \sigma^2 & : \quad h = 0 \\ 0 & : \quad h \geq 1 \end{cases} \tag{12}$$

$$\rho_z(h) = \frac{\gamma_z(h)}{\gamma_z(0)} = \begin{cases} 1 & : \quad h = 0 \\ 0 & : \quad h \geq 1 \end{cases} \tag{13}$$

b)

$$E(Z_t) = 0 \tag{14}$$

$$\gamma_z = \begin{cases} \sigma^2 & : \quad h = 0 \\ 0 & : \quad h \geq 1 \end{cases} \tag{15}$$

its weakly stationary.

7 a)

$$\begin{aligned} \gamma_x(h) \quad = \quad & cov(X_t, X_{t+h}) \\ & = cov(Z_t, Z_{t+h}) + \theta cov(Z_t, Z_{t+h-1} \\ & + \theta cov(Z_{t-1}, Z_{t+h}) + \theta^2 cov(Z_{t-1}, Z_{t+h-1}) \end{aligned} \tag{16}$$

$$\gamma_x(h) = \begin{cases} = 0 & : \quad \sigma_z^2 + \theta^2 \sigma_z^2 \\ = 1 & : \quad \theta \sigma_z^2 \\ \geq 2 & : \quad 0 \end{cases} \tag{17}$$

$$\rho_x(h) = \frac{\gamma_x(h)}{\gamma_x(0)} = \begin{cases} 1 & : \quad h = 0 \\ \frac{\theta}{1+\theta^2} & : \quad h = 1 \\ 0 & : \quad h \geq 2 \end{cases} \tag{18}$$

b) its weakly stationary