AMS316 Final in 2010 (2-hour exam)

Assume that z_t are independent and identically distributed normal random variables with mean 0 and variance σ^2.

1. Consider the process $x_t = a + bt + ct^2 + s_t + y_t$, in which s_t is a deterministic process satisfying $s_t = s_{t-12}$ and $s_t + \cdots + s_{t+11} = 0$, y_t is stationary process with mean zero.

   (a) What are the trend and seasonal components in the process x_t?

   (b) Let $s_t \equiv 0$. Is the lag-2 difference $(1-B)^2 X_t$ stationary?

2. Consider a stationary process $x_t = z_t + \theta z_{t-1} + \theta_2 z_{t-2}$.

   (a) What is the ACF of x_t at lag $k \geq 0$?

   (b) Suppose a new process y_t is expressed as $y_t - \alpha y_{t-1} = x_t$, where $|\alpha| < 1$. Find the ACF of y_t at lag k. (It is OK to write $\rho_y(k)$ as functions of $\rho_x(j)$)

3. Consider the AR(2) process:

$$x_t = \frac{1}{a(a+1)} x_{t-1} + \frac{1}{a(a+1)} x_{t-2} + z_t,$$

   in which the real number a is a parameter.

   (a) For what values of real number a, the process is stationary?

   (b) What is the autocorrelation of the process at lag k, i.e., $\rho(k)$? $(k \geq 0)$.

   (c) What are the 1-step and 2-step ahead forecasts of the process at forecast origin x_n?

4. Consider the stationary process $x_t + \phi x_{t-1} = z_t - \theta z_{t-1}$.

   (a) What are the stationarity and invertbility conditions for x_t?

   (b) Compute the lag-1 and lag-2 autocorrelations $\rho(1)$ and $\rho(2)$.

   (c) Given the observations $y_1, \ldots, y_n$, what are the 1- and 2-step ahead forecast of y at the forecast origin y_n?