**ASSIGNMENTS 3 OF AMS 516**

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**Ex 4.4 Proofs:** Since $\Sigma \sim IW_m(\Psi, n)$ and $W \sim W_m(\Sigma, n)$

We have $\pi(\Sigma|W) \propto \pi(\Sigma)f(W|\Sigma)$

$$= \frac{(\det\ \Psi)^{\frac{m}{2}}\left(\det(\Sigma)\right)^{-\frac{n_0+m+1}{2}} e^{-\frac{1}{2}tr(\Psi\Sigma^{-1})}}{2^{\frac{mn_0}{2}}\Gamma_m(\frac{n_0}{2})} * \frac{\det(W)^{\frac{n-m-1}{2}} e^{-\frac{1}{2}tr(\Sigma^{-1}W)}}{[2^m \det(\Sigma)]^{\frac{n}{2}}\Gamma_m(\frac{n}{2})}$$

$$= \frac{(\det\Psi)^{\frac{m}{2}} \det(W)^{\frac{n-m-1}{2}}}{\Gamma_m\left(\frac{n_0}{2}\right)\Gamma_m\left(\frac{n}{2}\right)} * \frac{\left(\det(\Sigma)\right)^{-\frac{n+n_0+m+1}{2}} \exp\{-\frac{1}{2}tr\left((\Psi+W)\Sigma^{-1}\right)\}}{2^{\frac{m(n+n_0)}{2}}}$$

$$\propto \frac{(\det(W+\Psi))^{\frac{m}{2}}\left(\det(\Sigma)\right)^{-\frac{n+n_0+m+1}{2}} \exp\left\{-\frac{1}{2}tr\left((\Psi+W)\Sigma^{-1}\right)\right\}}{2^{\frac{m(n+n_0)}{2}}\Gamma_m\left(\frac{n+n_0}{2}\right)}$$

$\sim IW_m(W+\Psi, n+n_0)$

**Ex 4.5 Proofs:** Since $\pi(\beta|\gamma) \sim N\left(z, \frac{V}{2\gamma}\right)$ and $\pi(\gamma) \sim gamma(g, \lambda)$

We have $\pi\left(\beta,\gamma\middle|(X,Y)\right) \propto f\left((X,Y)\middle|\beta,\gamma\right) * \pi(\beta,\gamma) = f\left((X,Y)\middle|\beta,\gamma\right) * \pi(\beta|\gamma) * \pi(\gamma)$

$$= \left(\frac{\gamma}{\pi}\right)^{\frac{n}{2}} e^{-\gamma(Y-X\beta)^T(Y-X\beta)} * (2\pi)^{-\frac{p}{2}}\left(\det\left(\frac{V}{2\gamma}\right)\right)^{-\frac{1}{2}} e^{-\gamma(\beta-z)^T V^{-1}(\beta-z)} * \frac{\lambda^g \gamma^{g-1} e^{-\lambda\gamma}}{\Gamma(g)}$$

Since $\left(\beta-\beta_n\right)^T V_n^{-1}\left(\beta-\beta_n\right) + \frac{1}{a_n}$

$$= \beta^T V^{-1}\beta + (X\beta)^T X\beta - z^T V^{-1}\beta - Y^T(X\beta) - \beta^T V^{-1} z - (X\beta)^T Y + \beta_n^T V_n^{-1}\beta_n + \frac{1}{a_n}$$

$$= (Y-X\beta)^T(Y-X\beta) + (\beta-z)^T V^{-1}(\beta-z) - z^T V^{-1} z - Y^T Y + \beta_n^T V_n^{-1}\beta_n + \frac{1}{a_n}$$

$$= (Y-X\beta)^T(Y-X\beta) + (\beta-z)^T V^{-1}(\beta-z) + \lambda$$

$$\pi\left(\beta,\gamma\middle|(X,Y)\right) = 2^{-\frac{p}{2}}\pi^{-\frac{n+p}{2}}\left(\det\left(\frac{V_n}{2\gamma}\right)\right)^{-\frac{1}{2}} e^{-\gamma\left(\beta-\beta_n\right)^T V_n^{-1}\left(\beta-\beta_n\right)} * \frac{\lambda^g \gamma^{g+\frac{n}{2}-1} e^{-\frac{\gamma}{a_n}}}{\Gamma(g)}$$

$$\propto (2\pi)^{-\frac{p}{2}}\left(\det\left(\frac{V_n}{2\gamma}\right)\right)^{-\frac{1}{2}} e^{-\gamma\left(\beta-\beta_n\right)^T V_n^{-1}\left(\beta-\beta_n\right)} * \frac{(1/a_n)^{g+\frac{n}{2}}\gamma^{g+\frac{n}{2}-1} e^{-\frac{\gamma}{a_n}}}{\Gamma\left(g+\frac{n}{2}\right)}$$

$= g(\beta|\gamma) * h(\gamma)$ Where $g(\beta|\gamma) \sim N\left(\beta_n, \frac{V_n}{2\gamma}\right)$, $h(\gamma) \sim gamma(g+\frac{n}{2}, \frac{1}{a_n})$

Hence $\pi\left(\beta\middle|\gamma\middle|(X,Y)\right) \sim N\left(\beta_n, \frac{V_n}{2\gamma}\right)$, $\pi\left(\gamma\middle|(X,Y)\right) \sim gamma(g+\frac{n}{2}, \frac{1}{a_n})$

**Ex 2.7 Solutions:**

(a) Sample covariance matrix $\widehat{V} = \begin{pmatrix} 0.2587 & 0.0758 & 0.0624 \\ 0.0758 & 0.3776 & 0.2367 \\ 0.0624 & 0.2367 & 0.3954 \end{pmatrix} * 10^{-3}$

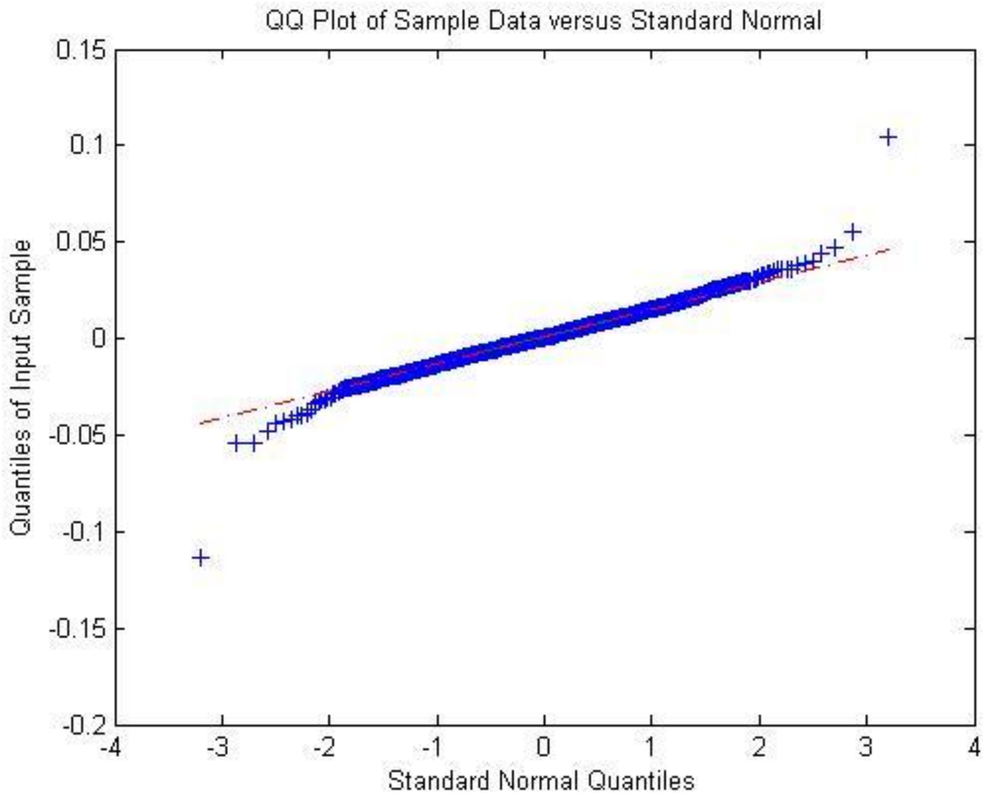
(b) From exercise 2.6(b), we have $\frac{\sqrt{n}(\hat{\rho}-\rho)}{1-\rho^2} \sim N(0,1)$

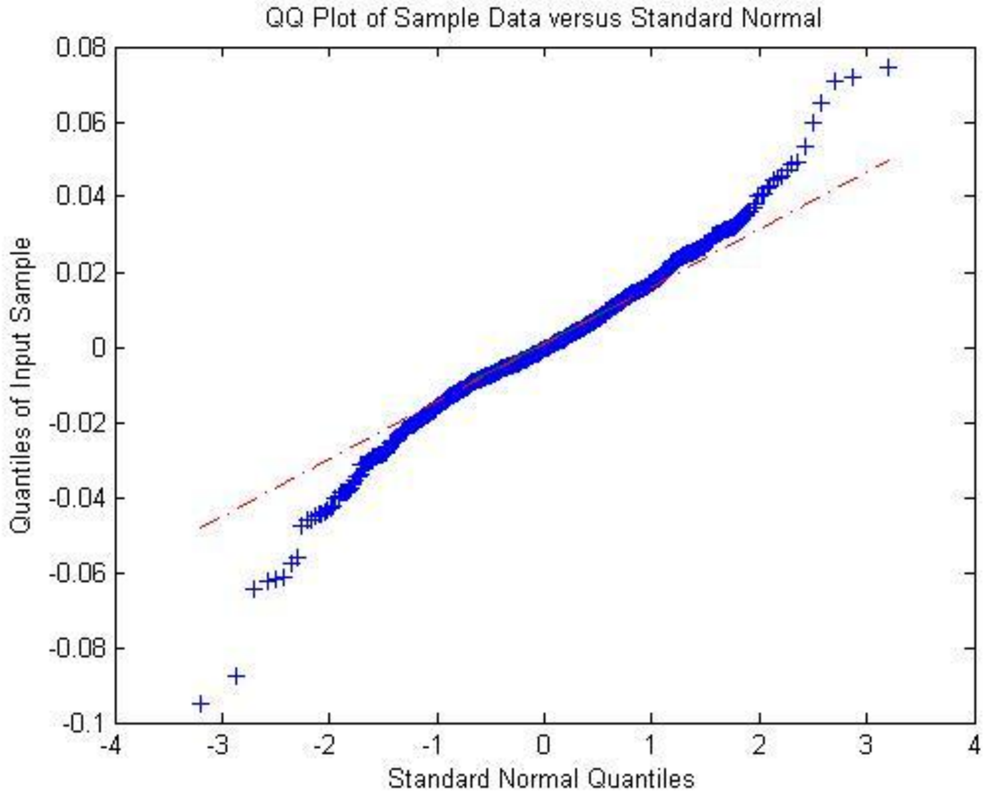
According to Slutsky's Theorem, we have $\frac{\sqrt{n}(\hat{\rho}-\rho)}{1-\hat{\rho}^2} \sim N(0,1)$

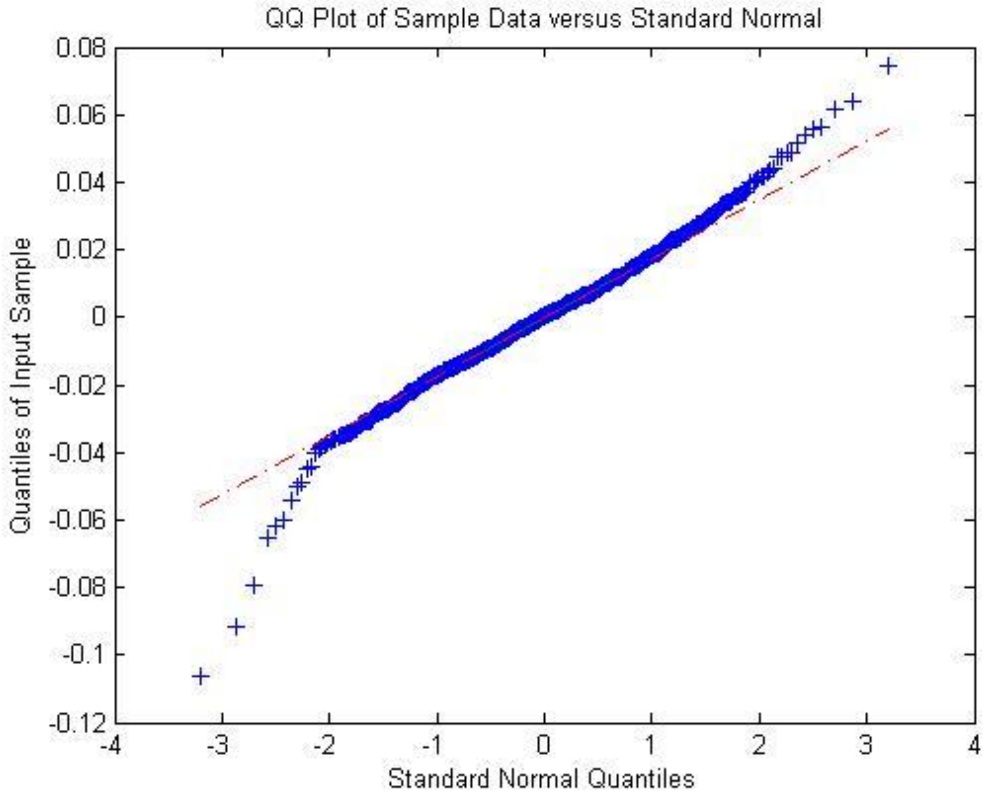
Hence the 95% C.I. for $\rho$ is $\hat{\rho} \pm Z_{0.025} * \frac{1-\hat{\rho}^2}{\sqrt{n}}$

| 95% C.I. for the correlation coefficient of the returns | |
|---|---|
| General & Ford | [0.1732, 0.3117] |
| Ford & Toyota | [0.5665, 0.6585] |
| Toyota & General | [0.1244, 0.2660] |

(c) In part (b), we make an assumption that the weekly log returns are i.i.d multivariate normal. We check the normality assumption with Q-Q plots with each manufacturer.





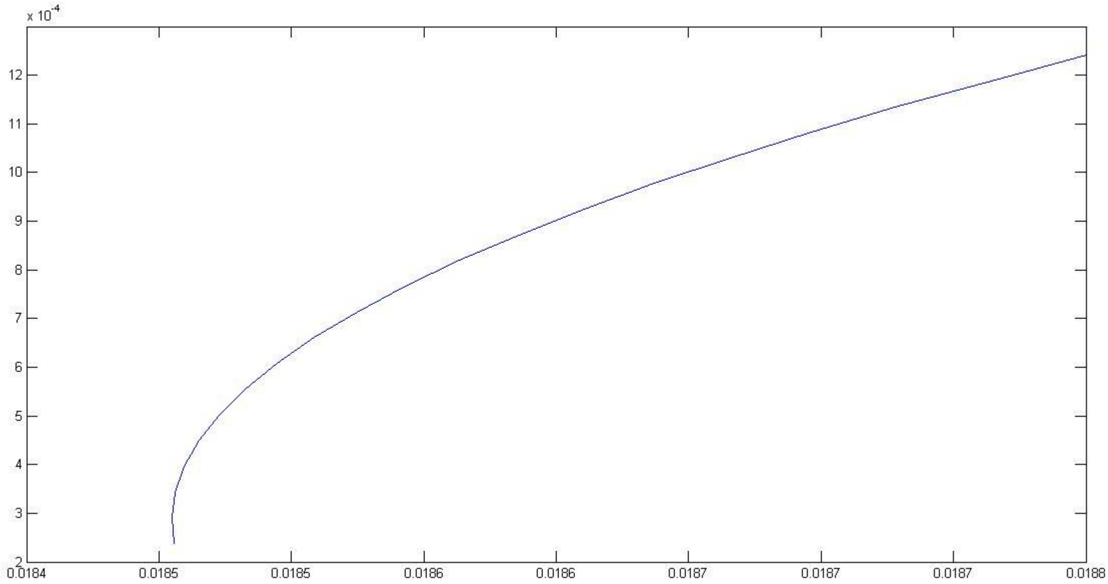

The three Q-Q plots are for General Motors Corp., Ford Motor Corp., and Toyota Motor Corp., respectively. The data are generally normal.

Matlab code:

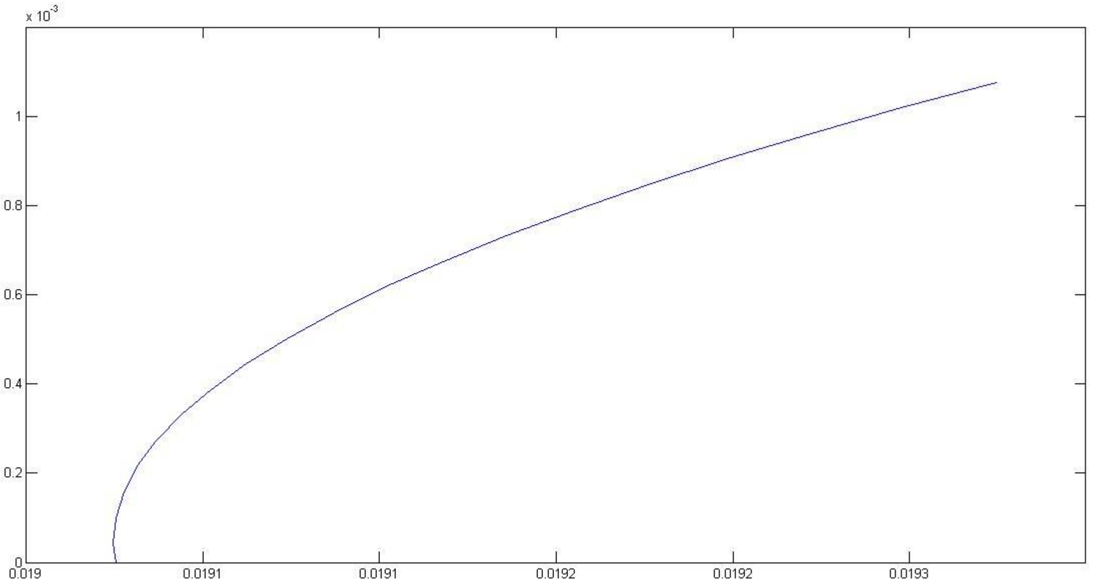
```
load w_logret_3automanu.txt;
Auto = w_logret_3automanu;
Cov = zeros(3);
for i = 1:3
    for j = 1:3
        Cov(i,j)=
(Auto(:,i+1)-mean(Auto(:,i+1)))'*(Auto(:,j+1)-mean(Auto(:,j+1)))/709;
    end;
end;
Corr = zeros(3);
for i = 1:3
    for j = 1:3
        Corr(i,j) = Cov(i,j)/sqrt(Cov(i,i)*Cov(j,j));
    end;
end;
CorrSd = zeros(3);
for i = 1:3
    for j = 1:3
        CorrSd(i,j) = (1-Corr(i,j)^2)/sqrt(709);
    end;
end;
Corr + CorrSd*norminv(0.975,0,1)
Corr - CorrSd*norminv(0.975,0,1)
qqplot(Auto(:,2))
qqplot(Auto(:,3))
qqplot(Auto(:,4))
```

**Ex 3.4 Solutions:**

(a) $B = 1$

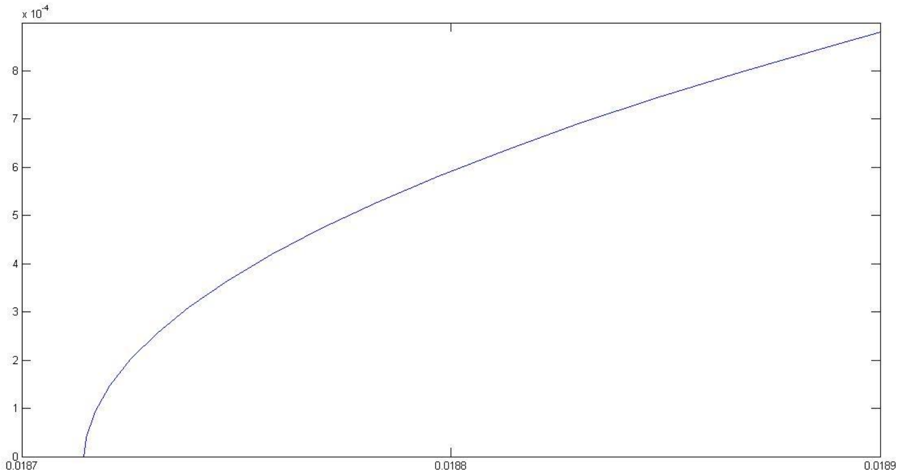



(b) B = 10




Since $\overline{w} = \frac{1}{B}\sum_{b=1}^{B} w_b^*$ is not the efficient portfolio given $\hat{\mu}$ and $\hat{\Sigma}$, the plotted curves are below the estimated efficient frontier based on $\hat{\mu}$ and $\hat{\Sigma}$.

(c) B = 500




Matlab code:

```
clear
load d_logret_6stocks.txt;
Stocks = d_logret_6stocks;
Cov = zeros(6);
M =mean(Stocks(:,2:7));
for i = 1:6
    for j = 1:6
        Cov(i,j)=
(Stocks(:,i+1)-mean(Stocks(:,i+1)))'*(Stocks(:,j+1)-mean(Stocks(:,j+1
)))/64;
    end;
end;
Rep = 500;
N = 64;
W = zeros(20,6);
for i = 1:Rep
    Resample = zeros(N,6);
    for j = 1:N
        for k = 1:6
            Resample(j,k) = Stocks(ceil(N*rand(1,1)),k+1);
        end
    end
    mu = mean(Resample);
    iSigma=inv(cov(Resample));
    ind=ones(1, 6);
    B = mu*iSigma*transpose(mu);
    A = ind*iSigma*transpose(mu);
    C = ind*iSigma*transpose(ind);
    D = B*C-A*A;
```

```
    for l = 1:20
        W(l,:) =
W(l,:)+transpose((B*iSigma*transpose(ind)-A*iSigma*transpose(mu)+0.00
01*l*(C*iSigma*transpose(mu)-A*iSigma*transpose(ind)))/D);
    end
end
W = W/Rep;
frontMu = W*M';
frontSd = zeros(1,20);
for l = 1:20
    frontSd(l) = sqrt(W(l,:)*Cov*W(l,:)');
end
plot(frontSd,frontMu)
xlim([1.87, 1.89]/100)
ylim([0, 0.9]/1000)
```