AMS 502 Final Exam Solutions

1. Proof.

$$u_t = F'(x+at)\cdot a + G'(x-at)(-a)$$

$$u_{tt} = F''(x+at)\cdot a^2 + G''(x-at)(-a)^2$$

$$u_x = F'(x+at) + G'(x-at)$$

$$u_{xx} = F''(x+at) + G''(x-at)$$

$$u_{tt} - a^2 u_{xx} = F''(x+at)\cdot a^2 + G''(x-at)\cdot a^2 - F''(x+at)\cdot a^2 - G''(x-at)\cdot a^2$$

$$= 0$$

$\square$

2. Solution. Step 1. Let $f_4(x) = e^{-x^2}$, we find $\hat{f}_4(\lambda)$.

$$\hat{f}_4(\lambda) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{-x^2} e^{-i\lambda x}\, dx$$

$$= \frac{1}{\sqrt{2\pi}} \frac{i}{\lambda} \left\{ \left[e^{-x^2} e^{-i\lambda x}\right]\Big|_{-\infty}^{+\infty} + 2\int_{-\infty}^{+\infty} x e^{-x^2} e^{-i\lambda x} dx \right\}$$

$$= \frac{2i}{\lambda}\left[x e^{-x^2}\right]^{\wedge} = \frac{-2}{\lambda} \frac{d}{d\lambda} \hat{f}_4(\lambda),$$

$$\frac{d\hat{f}_4}{d\lambda} = -\frac{\lambda}{2} \hat{f}_4$$

$$\hat{f}_4(0) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{-x^2} dx = \frac{1}{\sqrt{2}},$$

$$\hat{f}_4(\lambda) = \frac{1}{\sqrt{2}} e^{-\frac{\lambda^2}{4}}$$