The Invertible Matrix Theorem

Let A be a square $n \times n$ matrix. Then the following statements are equivalent. That is, for a given A, the statements are either all true or all false.

a. A is an invertible matrix.
b. A is row equivalent to the $n \times n$ identity matrix.
c. A has n pivot positions.
d. The equation $A\vec{x} = \vec{0}$ has only the trivial solution.
e. The columns of A form a linearly independent set.
f. The linear transformation $\vec{x} \to A\vec{x}$ is one-to-one. (*)
g. The equation $A\vec{x} = \vec{b}$ has at least one solution for each $\vec{b}$ in R^n.
h. The columns of A span R^n.
i. The linear transformation $\vec{x} \to A\vec{x}$ maps R^n onto R^n. (*)
j. There is an $n \times n$ matrix C such that $CA = I$.
k. There is an $n \times n$ matrix D such that $AD = I$.
l. A^T is an invertible matrix.
m. The columns of A form a basis of R^n.
n. $ColA = R^n$
o. $\dim ColA = n$
p. $rankA = n$
q. $NulA = \{\vec{0}\}$
r. $\dim NulA = 0$
s. The number 0 is not an eigenvalue of A.
t. The determinant of A is not zero.

(*) not covered in the class