

08/30/12

TA info

Herriman 010

Xi Zhang

Th

2-4pm

Tung Mai

W

12-2pm

Xiaohan Zhang

Tu

2-4pm

Homework #1

Due 09/11

1.1: 11, 14, 22

1.2: 4, 6a

1.3: 3ab, 5ab, 8

1.4: 1

2.1: 5ab, 12ac

## Section 1.3 Markov Chains

Static model

Dynamic model

A model that tries to predict the behavior of a system over a period of time.

A Markov Chain is a probabilistic model that describes the random movement over time of some activity.

At each period of time, the activity is in one of several possible states.

Example 1. Markov chain for Weather

Sunny cloudy  
Today

		Sunny	cloudy
Tomorrow	Sunny	$\frac{3}{4}$	$\frac{1}{2}$
	cloudy	$\frac{1}{4}$	$\frac{1}{2}$

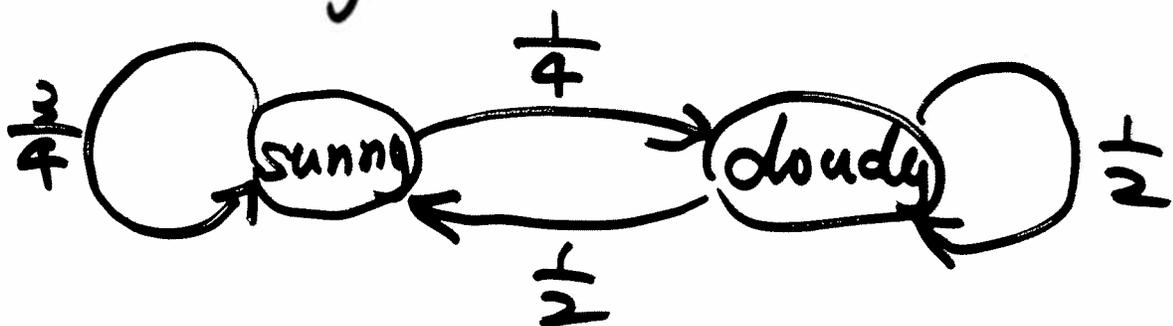
transition matrix

transition probabilities

$$\begin{bmatrix} \frac{3}{4} & \frac{1}{2} \\ \frac{1}{4} & \frac{1}{2} \end{bmatrix}$$

each column should add up to 1.

transition diagram



Notation: probability distribution

let  $p_1, p_2, \dots, p_n$  be the probabilities of being in state 1, 2, ..., n.

let  $a_{ij}$  be the transition probability of going from state  $j$  to state  $i$ .

Weather            1 sunny  
                          2 cloudy

$$a_{11} = \frac{3}{4}, \quad a_{12} = \frac{1}{2}$$

$$a_{21} = \frac{1}{4}, \quad a_{22} = \frac{1}{2}$$

Formula for Distribution of Next states in Markov Chain.

$p_1, p_2, \dots, p_n,$

$p'_1, p'_2, \dots, p'_n$  for the next period

$$p'_1 = a_{11}p_1 + a_{12}p_2 + a_{13}p_3 + \dots + a_{1n}p_n$$

$$p'_2 = a_{21}p_1 + a_{22}p_2 + a_{23}p_3 + \dots + a_{2n}p_n$$

$\vdots$

$$p'_n = a_{n1}p_1 + a_{n2}p_2 + a_{n3}p_3 + \dots + a_{nn}p_n$$

Weather model.

$$a_{11} = \frac{3}{4} \quad a_{12} = \frac{1}{2}$$
$$a_{21} = \frac{1}{4} \quad a_{22} = \frac{1}{2}.$$

$$P_1' = a_{11}P_1 + a_{12}P_2$$

$$P_2' = a_{21}P_1 + a_{22}P_2.$$

$$P_1 = \frac{3}{4} \quad P_2 = \frac{1}{4}$$

$$P_1' = \frac{3}{4} \times \frac{3}{4} + \frac{1}{2} \times \frac{1}{4} = \frac{11}{16}$$

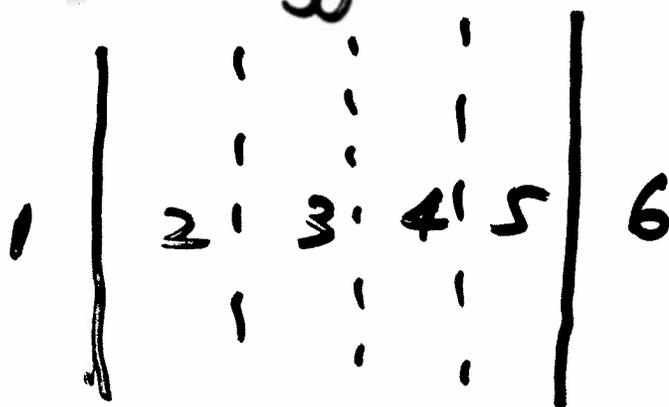
$$P_2' = \frac{1}{4} \times \frac{3}{4} + \frac{1}{2} \times \frac{1}{4} = \frac{5}{16}$$

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$$P_1 = \frac{11}{16}, \quad P_2 = \frac{5}{16} \quad \text{tomorrow}$$

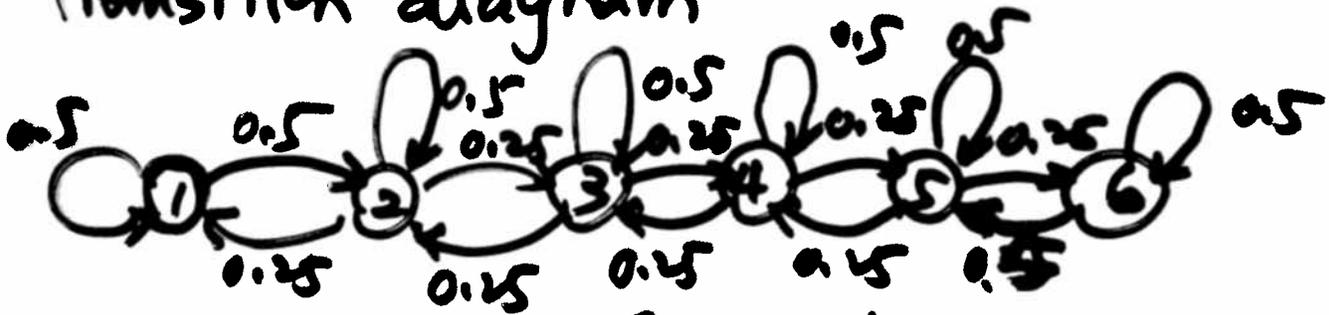
$P_1'$   
 $P_2'$  day after tomorrow

Example 2. Frogger Markov Chain



Transition Matrix.

Transition Diagram



Current

	1	2	3	4	5	6
Next	0.5	0.25	0	0	0	0
0.5	0.5	0.25	0	0	0	0
0	0	0.25	0.5	0.25	0	0
0	0	0	0.25	0.5	0.25	0
0	0	0	0	0.25	0.5	0.5
0	0	0	0	0	0.25	0.5

$$P_1' = 0.5P_1 + 0.5P_2$$

$$P_2' = 0.5P_1 + 0.5P_2 + 0.25P_3$$

$$P_3' = 0.25P_2 + 0.5P_3 + 0.25P_4$$

⋮

$$0.5P_5 + 0.5P_6$$

$P_6'$

Initially

$$P_1 = 1, P_2 = \dots = P_6 = 0.$$

$$P_1' = 0.5$$

$$P_2' = 0.5, P_3' = \dots = P_6' = 0$$

after 1 period of time

after a period of time

$$P_1' = 0.375 \cdot P_2' = 0.5 \cdot P_3' = 0.125$$

$$P_4' = P_5' = P_6' = 0.$$

Section 1.4, Linear programming  
and Models without Exact solutions.

# of unknowns

# of equation

Oil Refinery Model

Section 1.2.

Heating oil  $20X_1 + 4X_2 + 4X_3 = 500$

diesel oil  $10X_1 + 14X_2 + 5X_3 = 850$

gasoline  $5X_1 + 5X_2 + 12X_3 = 1000$

suppose refinery 3 break down

$$20X_1 + 4X_2 = 500$$

$$10X_1 + 14X_2 = 850$$

$$5X_1 + 5X_2 + \cancel{12X_3} = 1000.$$

More equations than unknowns.

— overdetermined

\* not normally have a solution.

Ex. Refinery Model with Diesel oil

$$20x_1 + 4x_2 + 4x_3 = 500$$

$$5x_1 + 5x_2 + 12x_3 = 1000,$$

more unknowns than equations.

— underdetermined.

several possible solutions.

cost of operating refinery.

#1      \$30 per barrel

#2      \$25

#3      \$20

minimize the cost

$$30x_1 + 25x_2 + 20x_3.$$

operation cost

↑  
minimize

constraints

$$20x_1 + 4x_2 + 4x_3 = 500$$

$$5x_1 + 5x_2 + 12x_3 = 1000$$

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$$

optimization.

# Linear programming

the problem of optimizing (minimizing or maximizing) a linear expression subject to constraints that are linear equations or inequalities.

overdetermined  
underdetermined.

50 equations. 2 unknowns.

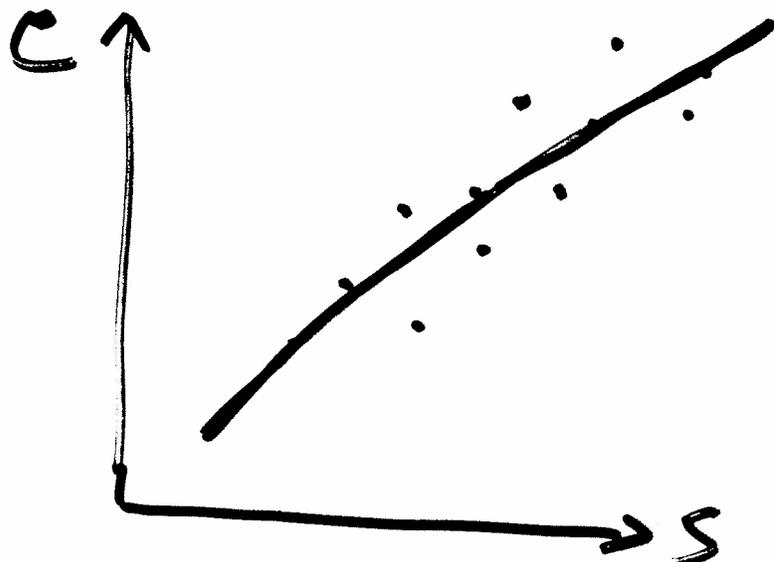
regression.

Ex. Predicting grades.

College GPA =  $\beta \times$  (high school GPA) +

$$C = \beta \times S + r.$$

$\beta, r$ . constants to be determined.



regression