1. (10 points)

a) (5 points) Give a definition of "basis of a vectorspace" .

b) (5 points) Are the columns of $\begin{bmatrix} 0 & 0 & 4 \\ 0 & 1 & 3 \\ 2 & 0 & 4 \end{bmatrix}$ a basis for $\mathbb{R}^3$? (Explain your answer).

2. (10 points)

a) (5 points) Use determinants to determine if the matrix $\begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 1 \\ 3 & 2 & 0 \end{bmatrix}$ is invertible.

b) (5 points) Use determinants to determine if the matrix $\begin{bmatrix} 1 & 2 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 2 & 0 & 2 \\ 4 & 2 & 1 & 2 \end{bmatrix}$ is invertible.

3. (10 points) Find the coordinates of the vector $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ relative to the basis $\{ \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 3 \\ 4 \end{bmatrix} \}$.

4. (10 points) A is the matrix $\begin{bmatrix} 3 & 2 & -1 \\ 2 & 1 & 1 \\ 1 & 0 & 3 \end{bmatrix}$

a) (5 points) Find a basis for the nullspace of A.

b) (5 points) Find a basis for the columnspace of A.

5. A is the matrix $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ and W is the subspace of 2×2 matrices B for which

$$BA = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

a) (6 points) Find a basis for W.

b) (4 points) Find the dimension of W.

6. (10 points)

P_2 is the vectorspace of polynomials of degree less than or equal to 2.

Define $T : P_2 \to \mathbb{R}^3$ by $T(p) = \begin{bmatrix} p(-1) \\ p(0) \\ p(1) \end{bmatrix}$.

a) (2 points) Let $p(t) = 3 + 5t + t^2$, determine $T(p)$.

b) Is T invertible? (carefully explain your answer).