3.5 \( A=\{3,4\}, \ B=\{2,3\}, \ C=\{4,5\}. \)

(a) \( A \cup B=\{2,3,4\} \). Work is easy, average or difficult on this model.

(b) \( A \cap B=\{3\} \). Work is average on this model.

(c) \( \overline{B}=\{1,4,5\} \). Thus \( A \cup \overline{B}=\{1,3,4,5\} \). Work is not easy on this model.

(d) \( \overline{C}=\{1,2,3\} \). Work is very easy, easy or average on this model.

3.13 The following Venn diagram will be used in parts (a), (b), (c) and (d).

(a) \( A \cap B \) is region 2 in Fig. 1. \( (A \cap B) \) is the region composed of areas 1, 3, and 4. \( \overline{A} \) is the region composed of areas 3 and 4. \( \overline{B} \) is the region composed of areas 1 and 4. \( \overline{A} \cup \overline{B} \) is the region composed of areas 1, 3, and 4. This corresponds to \( (A \cap \overline{B}) \).

(b) \( A \cap B \) is the region 2 in the figure. \( A \) is the region composed of areas 1 and 2. Since \( A \cap B \) is entirely contained in \( A \), \( A \cup (A \cap B) = A \).

(c) \( A \cap B \) is region 2. \( A \cap \overline{B} \) is region 1. Thus, \( (A \cap B) \cup (A \cap \overline{B}) \) is the region composed of areas 1 and 2 which is \( A \).

(d) From part (c), we have \( (A \cap B) \cup (A \cap \overline{B}) = A \). Thus, we must show that \( (A \cap B) \cup (A \cap \overline{B}) \cup (\overline{A} \cap B) = A \cup (\overline{A} \cap B) = A \cup B \). \( A \) is the region composed of areas 1 and 2 and \( \overline{A} \cap B \) is region 3. Thus, \( A \cup (\overline{A} \cap B) \) is the region composed of areas 1, 2, and 3.

(e) In Fig. 2, \( A \cup B \) is the region composed of areas 1, 2, 3, 4, 5, and 6. \( A \cup C \) is the region composed of areas 1, 2, 3, 4, 6, and 7, so \( (A \cup B) \cap (A \cup C) \) is the region composed of areas 1, 2, 3, 4, and 6. \( B \cap C \) is the region composed of areas 3, and 6, and \( A \) is the region composed of areas 1, 2, 3, and 4. Thus, \( A \cup (B \cap C) \) is the region composed of areas 1, 2, 3, 4, and 6. Thus \( A \cup (B \cap C) = (A \cup B) \cap (A \cup C) \).
3.19 (a) \( P_4 \approx 8!/4! = 1,680. \)
(b) \( 8^4 = 4,096. \)

3.21 \( 6! = 720. \)

3.22 (a) There are \( 5!/2! = 60 \) permutations.
(b) There are 6 commercials, 3 of which are alike. Thus, there are \( 6!/3! = 120 \) ways to fill the time slots.

3.30 There are 250 numbers divisible by 200. Thus, the probability is \( 250/50,000 = 1/200. \)

3.38 (a) \(.38 \) and \(.53 \) do not sum to 1.
(b) Probability cannot be negative.
(c) The probability that the compressor or the fan motor is all right is \(.82 + .64 - .41 = 1.05 \) which is greater than 1.

3.43 (a) This probability is given by \(.22 + .21 = .43. \)
(b) \(.17 + .29 + .21 = .67. \)
(c) \(.03 + .08 = .11. \)
(d) \(.22 + .29 + .08 = .59. \)

3.47 (a) “At least one award” is the same as “design or efficiency award”. Thus, the probability is \(.16 + .24 - .11 = .29. \)
(b) This the probability of “at least one award” minus the probability of both awards or \(.29 - .11 = .18. \)

3.48 (a) \( P(A \cup B) = P(A) + P(B) - P(A \cap B) = .35 + .73 - .14 = .94. \)
(b) \( P(A \cap B) = P(B) - P(A \cap B) = .73 - .14 = .59. \)
(c) \( P(A \cap \overline{B}) = P(A) - P(A \cap B) = .35 - .14 = .21. \)
(d) \( P(A \cup \overline{B}) = P(A \cap \overline{B}) = 1 - P(A \cap B) = 1 - .14 = .86. \)