4.2 This problem satisfies the binomial assumptions with \( n = 4 \) and \( p = 1/2 \). Thus the distribution is

\[
P(\text{Total number of heads}) = \binom{4}{x} \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{4-x} = \binom{4}{x} \left(\frac{1}{2}\right)^4
\]

for \( x = 0, 1, 2, 3, 4 \). Therefore

\[
P(0) = 1/16, \quad P(1) = 1/4, \quad P(2) = 6/16, \quad P(3) = 1/4, \quad P(4) = 1/16.
\]

4.5 Using the identity

\[
(x - 1) \sum_{i=0}^{n} x^i = x^{n+1} - 1
\]

or

\[
\sum_{i=0}^{n} x^i = \frac{x^{n+1} - 1}{x - 1},
\]

we have

\[
\sum_{x=0}^{4} \frac{k}{2^x} = k \frac{\left(\frac{1}{2}\right)^{4+1} - 1}{\frac{1}{2} - 1} = \frac{31k}{16}.
\]

This must equal 1, so \( k = 16/31 \).

4.18 (a) \( b(1; 12, .05) = B(1; 12, .05) - B(0; 12, .05) = (.8816) - (.5404) = .3412 \).

(b) \( B(2; 12, .05) = .9804 \).

(c) \( 1 - B(1; 12, .05) = 1 - (.8816) = .1184 \).

4.20 (a) The probability that 1 or more components in a sample of 15 is defective when the true probability of being good is .95 is

\[
1 - b(0; 15, .05) = 1 - (.95)^{15} = .5367.
\]

(b) The probability that 0 are defective when the true probability is .90 is \( b(0; 15, .10) = .2059 \).

(c) When the true probability is .90, we have \( b(0; 15, .20) = .0352 \).
The probabilities are:

<table>
<thead>
<tr>
<th>i</th>
<th>( b(i; 10, .7) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>.0000</td>
</tr>
<tr>
<td>1</td>
<td>.0001</td>
</tr>
<tr>
<td>2</td>
<td>.0014</td>
</tr>
<tr>
<td>3</td>
<td>.0090</td>
</tr>
<tr>
<td>4</td>
<td>.0368</td>
</tr>
<tr>
<td>5</td>
<td>.1029</td>
</tr>
<tr>
<td>6</td>
<td>.2001</td>
</tr>
<tr>
<td>7</td>
<td>.2668</td>
</tr>
<tr>
<td>8</td>
<td>.2335</td>
</tr>
<tr>
<td>9</td>
<td>.1211</td>
</tr>
<tr>
<td>10</td>
<td>.0282</td>
</tr>
</tbody>
</table>

The probability histogram is given in Figure 4.1.

4.36 Using the formula

\[
\sum_{i=0}^{n} i = \frac{1}{2} n(n + 1)
\]

and

\[
\sum_{i=0}^{n} i^2 = \frac{1}{6} n(n + 1)(2n + 1)
\]

we find

\[
\mu = \frac{1}{n} \sum_{i=0}^{n} i = \frac{n + 1}{2}
\]

and

\[
\sigma^2 = \mu_2 - \mu^2 = \frac{1}{n} \frac{n(n + 1)(2n + 1)}{6} - \left( \frac{n + 1}{2} \right)^2
\]

\[
= \frac{(n + 1)(n - 1)}{12} = \frac{(n^2 - 1)}{12}.
\]

4.37 (a) The mean for the binomial distribution with \( n = 4 \) and \( p = 7 \) can be calculated from the following table:

<table>
<thead>
<tr>
<th>( i )</th>
<th>( b(i; 4, .7) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>.0081</td>
</tr>
<tr>
<td>1</td>
<td>.0756</td>
</tr>
<tr>
<td>2</td>
<td>.2646</td>
</tr>
<tr>
<td>3</td>
<td>.4116</td>
</tr>
<tr>
<td>4</td>
<td>2401</td>
</tr>
</tbody>
</table>

Thus,

\[
\mu = 0(.0081) + 1(.0756) + 2(.2646) + 3(.4116) + 4(.2401) = 2.8
\]

\[
\mu_2 = 0^2(.0081) + 1^2(.0756) + 2^2(.2646) + 3^2(.4116) + 4^2(.2401) = 8.68
\]

\[
\sigma^2 = 8.68 - (2.8)^2 = .84.
\]
4.40 (a) First we need to find \( b(i; 20, .95) \), \( i = 0 \) to 20. These are given in the following table:

<table>
<thead>
<tr>
<th>( i )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( b(i; 20, .95) )</td>
<td>.0000</td>
<td>.0000</td>
<td>.0000</td>
<td>.0000</td>
<td>.0000</td>
<td>.0000</td>
<td>.0000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( i )</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
</tr>
</thead>
<tbody>
<tr>
<td>( b(i; 20, .95) )</td>
<td>.0000</td>
<td>.0000</td>
<td>.0000</td>
<td>.0000</td>
<td>.0000</td>
<td>.0000</td>
<td>.0000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( i )</th>
<th>14</th>
<th>15</th>
<th>16</th>
<th>17</th>
<th>18</th>
<th>19</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>( b(i; 20, .95) )</td>
<td>.0003</td>
<td>.0022</td>
<td>.0133</td>
<td>.0596</td>
<td>.1887</td>
<td>.3773</td>
<td>.3585</td>
</tr>
</tbody>
</table>

Thus,

\[
\mu = (0 + 1 + \cdots + 13)(.0000) + 14(.0003) + 15(.0022) + 16(.0133) \\
+17(.0596) + 18(.1887) + 19(.3773) + 20(.3585) = 19.0000
\]

and

\[
\mu'_2 = (0^2 + 1^2 + \cdots + 13^2)(.0000) + 14^2(.0003) + 15^2(.0022) + \\
16^2(.0133) + 17^2(.0596) + 18^2(.1887) + 19^2(.3773) + 20^2(.3585) \\
= 361.9496
\]

Therefore,

\[
\sigma^2 = 361.9496 - (19.0000)^2 = .9496.
\]

(b) The special formulas for the binomial give:

\[
\mu = np = 20(.95) = 19
\]

\[
\sigma^2 = np(1 - p) = 20(.95)(.05) = .95
\]
The tail probabilities and upper bounds are

<table>
<thead>
<tr>
<th>No. of sd's</th>
<th>Upper bound of tail probabilities</th>
<th>Tail probabilities from binomial (16; 1/2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>.4544</td>
</tr>
<tr>
<td>2</td>
<td>1/4</td>
<td>.0768</td>
</tr>
<tr>
<td>3</td>
<td>1/9</td>
<td>.0042</td>
</tr>
</tbody>
</table>

The upper bound of the tail probability comes from

\[ P(|X - \mu| \geq k\sigma) \leq \frac{1}{k^2} \]

where \( k \) is the number of standard deviations. An example of the calculation of the tail probabilities for the binomial distribution with \( n = 16 \) and \( p = 1/2 \) follows for the case \( k = 2 \),

\[ \mu = np = 8, \quad \sigma^2 = np(1 - p) = 4, \quad \sigma = 2. \]

Thus, we need \( P(X \leq 4) + P(X \geq 12) \) when \( X \) has distribution \( b(x; 16, 1/2) \).

From Table 1, this is \( .0384 + 1 - .9616 = .0768 \).

4.48 We need to find \( k \) such that \( 1/k^2 = .01 \). Thus, \( k = 10 \). Chebyshev's theorem states that,

\[ P(|X - \mu| \geq k\sigma) \leq \frac{1}{k^2}, \]

or

\[ P(\frac{X}{n} - \frac{\mu}{n} \geq k\frac{\sigma}{n}) \leq \frac{1}{k^2}, \]

Now \( \mu = .5n, \sigma^2 = n/4, \sigma = \sqrt{n}/2 \). Thus, we need to find \( n \) such that,

\[ k\frac{\sigma}{n} = .04 \quad \text{or} \quad \frac{10\sqrt{n}/2}{n} = .04, \]

or

\[ n = \left(\frac{5}{.04}\right)^2 = 15,625. \]