1. **Null hypothesis** $H_0: \mu_1 - \mu_2 = 30$

   **Alternative hypothesis** $H_1: \mu_1 - \mu_2 > 30$

2. **Level of significance:** $\alpha = 0.01$.

3. **Criterion:** The null hypothesis specifies $\delta = \mu_1 - \mu_0 = 30$. Since the samples are large, we use the large sample statistic where we estimate each population variance by its sample variance

   $$Z = \frac{\bar{X}_1 - \bar{X}_2 - \delta}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$$

   The alternative is one-sided so we reject the null hypothesis for $Z > z_{.01} = 2.33$

4. **Calculations:** Since $n_1 = 60$, $n_2 = 60$, $\bar{x}_1 = 585.00$, $\bar{x}_2 = 532.20$, $s_1 = 31.20$, and $s_2 = 36.40$

   $$Z = \frac{585.00 - 532.20 - 30}{\sqrt{31.20^2/60 + 36.40^2/60}} = 3.68$$

5. **Decision:** Because $3.68 > 2.33$, we reject the null hypothesis at the .01 level of significance. Men earn an average of more than thirty dollars per week than the women.

   The P-value $P[Z > 3.68]$, about .0002 gives even stronger support for rejecting the null hypothesis.
7.66 (a) 1. Null hypothesis \( H_0: \mu_1 - \mu_2 = 0 \)

Alternative hypothesis \( H_1: \mu_1 - \mu_2 \neq 0 \)

2. Level of significance: \( \alpha = 0.01 \).

3. Criterion: The null hypothesis specifies \( \delta = \mu_1 - \mu_2 = 0 \). Since the samples are large, we use the large sample statistic where we estimate each population variance by its sample variance

\[
Z = \frac{\bar{X}_1 - \bar{X}_2 - \delta}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}
\]

The alternative is two-sided so we reject the null hypothesis for \( Z > z_{0.005} \) or \( Z < -z_{0.005} \) where \( z_{0.005} = 2.58 \).

4. Calculations: Since \( n_1 = 40, n_2 = 30, \bar{x}_1 = 247.3, \bar{x}_2 = 254.1, s_1 = 15.2, \) and \( s_2 = 18.7 \)

\[
z = \frac{247.3 - 254.1}{\sqrt{15.2^2/60 + 18.7^2/30}} = -1.62
\]

5. Decision: At the .05 level of significance, we cannot reject the null hypothesis of no difference in mean number of cars.

(b) Since the alternative hypothesis is two-sided with \( \alpha = .01 \), we use Table 8d.
In this case,
\[ d = \frac{15.6}{\sqrt{15.2^2 + 18.7^2}} = .647 \]

We take \( n \) to be given by
\[ n = \frac{15.2^2 + 18.7^2}{15.2^2/40 + 18.7^2/30} = 33.3 \approx 34 \]

Thus, the probability of a Type II error is between .25 and .35.

7.71

1. **Null hypothesis** \( H_0 : \mu = 0 \)

   **Alternative hypothesis** \( H_1 : \mu \neq 0 \)

2. **Level of significance**: \( \alpha = 0.05 \).

3. **Criterion**: The number of pairs is small so we must assume that each difference has a normal distribution. We use the paired \( t \) statistic

   \[ t = \frac{D_1 - \delta}{S_D/\sqrt{n}} \]

   Since the alternative hypothesis is two-sided, we reject the null hypothesis if \( |t| > t_{.025} = 2.262 \).

![t-distribution graph](image)

4. **Calculations**: The difference between the weights on Scale I and the weights on Scale II are

   \[ -.04, -.05, -.02, -.02, .01, -.02, -.05, .01, -.05, .03 \]

   The mean of this sample is \( -.02 \) and the variance is \( s^2 = .0008 \). The null distribution specifies \( \delta = 0 \) so

   \[ t = \frac{- .02 - 0}{\sqrt{.0008/9}} = -2.12 \]

5. **Decision**: We cannot reject the null hypothesis at level of significance .05.
1. Null hypothesis $H_0 : \mu = 0$
   
   Alternative hypothesis $H_1 : \mu > 0$

2. Level of significance: $\alpha = 0.01$.

3. Criterion: The number of pairs is moderately small so we must assume that each difference has a normal distribution. We use the paired $t$ statistic

   \[ t = \frac{\bar{D} - \delta}{SD/\sqrt{n}} \]

   Since $\alpha = .01$ and the alternative hypothesis is one-sided, we reject the null hypothesis if $t > t_{.01}$. There are 15 degrees of freedom so $t > t_{.01} = 2.602$.

4. Calculations: The sample mean of the differences is 4.0625 and the variance is 16.996.

   \[ t = \frac{4.0625 - 0}{\sqrt{16.996/16}} = 3.94 \]

5. Decision: We reject the null hypothesis at level of significance .01. Thus, the physical exercise program is effective. The $P-$value is less than .005 so the evidence against the null hypothesis is strong.

(a) Rejection region  (b) P-value for Problem 7.72
9.9 The sample proportion is $204/300 = .68$. Using the large sample confidence interval with $z_{\alpha/2} = 2.33$ gives

$$.68 - 2.33\sqrt{\frac{(.68)(.32)}{300}} < p < .68 + 2.33\sqrt{\frac{(.68)(.32)}{300}}$$

or $.617 < p < .743$ as the 98% confidence interval.

9.17 The sample proportion is $533/4063 = .131$. Using the large sample confidence interval with $z_{\alpha/2} = 1.96$ gives

$$.131 - 1.96\sqrt{\frac{(.131)(.869)}{4063}} < p < .131 + 1.96\sqrt{\frac{(.131)(.869)}{4063}}$$

or $.12 < p < .14$ as the 95% confidence interval.

9.19 1. **Null hypothesis** $H_0 : p = .3$

**Alternative hypothesis** $H_1 : p > .3$

2. **Level of significance**: $\alpha = 0.05$.

3. **Criterion**: Using a normal approximation for the binomial distribution, we reject the null hypothesis when

$$Z = \frac{X - np_0}{\sqrt{np_0(1 - p_0)}} > z_{.05}.$$ 

Since $\alpha = .05$ and $z_{.05} = 1.645$, the null hypothesis must be rejected if $Z > 1.645$.

4. **Calculations**: $p_0 = .3$, $X = 47$, and $n = 120$ so

$$Z = \frac{47 - 120(.30)}{\sqrt{120(.30)(.70)}} = 2.19.$$ 

5. **Decision**: Since the observed value $2.19 > z_{.05} = 1.645$, we reject the null hypothesis at the 5% level of significance. The evidence against the null hypotheses is quite strong since the P-value is .0143.
9.20 1. Null hypothesis $H_0 : p = .2$
    
    Alternative hypothesis $H_1 : p \neq .2$

2. Level of significance: $\alpha = 0.05$.

3. Criterion: Using a normal approximation for the binomial distribution, we reject the null hypothesis when

   $$Z = \frac{X - np_0}{\sqrt{np_0(1-p_0)}} < -z_{0.025} \text{ or } Z > z_{0.025}.$$  

Since $\alpha = 0.05$ and $z_{0.025} = 1.96$, the null hypothesis must be rejected if $|Z| > 1.96$.

4. Calculations: $p_0 = .2$, $X = 11$, and $n = 104$ so

   $$Z = \frac{11 - 104(.2)}{\sqrt{104(.2)(.8)}} = -2.40.$$  

5. Decision: Since the observed value $-2.40 < -z_{0.025} = -1.96$, we reject the null hypothesis at the 5% level of significance. The evidence against the null hypothesis is quite strong since the P-value is .0164.
9.24 Since the sample size is small, we cannot use the large sample statistic. In this case, the null hypothesis is \( p = .60 \), the alternative is \( p < .60 \), and the significance level is .05. Thus, we need to find the largest value \( x_\alpha \) such that the probability of \( x_\alpha \) or fewer successes in a sample of size 13 when \( p = .60 \) is less than or equal to .05. We then reject the null hypothesis when \( X \leq x_\alpha \). Using Table 1 we see that \( x_\alpha = 5 \). Since \( X = 4 \), we reject the null hypothesis.

9.25 The null hypothesis is \( p = .50 \), the alternative is \( p \neq .50 \), and the significance level is .05. Thus, we need to find two values \( x_{\alpha_1} \) and \( x_{\alpha_2} \) where the probability of \( x_{\alpha_1} \) or fewer heads plus the probability of \( x_{\alpha_2} \) or more heads in 15 flips, when \( p = .50 \), is less than or equal to .05. Using Table 1 we see that \( x_{\alpha_1} = 3 \) and \( x_{\alpha_2} = 12 \) satisfy

\[
P[ X \leq 3 \text{ or } X \geq 12 \mid n = 15, p = .50 ] = .0176 + (1 - .9824) = .0352.
\]

The actual level of significance is then .0352.