# AMS 326 - Final Project

## Due 5 pm, Monday, December 18, 2006

**1.** (10 points) **(Bouncing Ball Problem)**
Consider the equation

$$y'' + b|y'|y' + g = q(y) \text{ on } (0, 2).$$

Suppose that $y(0) = 1$, $y'(0) = 0$, $g = 10$. The function $q(y)$ is zero for y nonnegative and $-10^6 y$ for y negative.
If we let $E(t) = gy + 0.5(y')^2 + Q(y)$, where $Q(y)$ is zero for y nonnegative and $5 * 10^5 * y^2$ for y negative. Then

$$E' = -b|y'|^3.$$

Using $Y = (y, y', E)^T$, the above relations can be expressed as an initial value problem. Use Heun's method to solve this problem.
Implement for time step size $h = 0.0001$ and $b = 0.01, 0.001, 0.0001$.
Produce three plots $(y, t)$, $(y', t)$, and $(E, t)$ on $(0, 2)$ for the above $h$ and $b$.

**2.** (10 points) Since

$$\int_0^1 \frac{4}{1+x^2} dx = \pi,$$

one can compute an approximate value for $\pi$ using numerical integration of the given function.
Use the midpoint, trapezoid, and Simpson composite quadrature rules to compute the approximate value for $\pi$ for numbers of subintervals 2, 4, 8, 16, 32, 64, 128, 256. Compute the errors and order of accuracy.

**3.** (5 points) The temperature dependence of reaction rate coefficient of a chemical reaction is often modeled by the Arrhennius equation

$$k = Aexp(-E_a/RT),$$

where $k$ is the reaction rate, $A$ is the preexponential factor, $E_a$ is the activation energy, $R$ is the universal gas constant, and $T$ is the absolute temperature. Experimental data for a particular reaction yield the following results: Use a least-squares fit of this data to obtain values for $A$ and

| T | 773.5 | 786 | 797.5 | 810 | 810 | 820 | 834 |
|---|---|---|---|---|---|---|---|
| k | 1.63 | 2.95 | 4.19 | 8.13 | 8.19 | 14.9 | 22.2 |

$E_a$ for the reaction. Take R = 8314. Plot the least square fit with solid line and data with "$\circ$".