Instructions.
This is a close book and close notes exam. You can use one piece (8 x 11) of cheat sheet. Please provide detailed solutions. Anyone who cheats on the exam shall receive a grade of F.

1. The city housing department wants to estimate the average rent for rent-controlled apartments. They need to determine the number of renters to include in the survey in order to estimate the average rent to within $50 using a 95% confidence interval. From past results, the rent for controlled apartments ranged from $200 to $1500 per month. How many renters are needed in the survey to meet the requirements?

Answer: \[
\sigma = \frac{1500 - 200}{4} = 325, \quad E = 50, \quad \alpha = 0.05 \Rightarrow n = \frac{1.96^2 \cdot 325^2}{50^2} = 163,
\]
Therefore, 163 renters are needed in the survey to meet the requirements.

2. To evaluate the success of a 1-year experimental program designed to increase the mathematical achievement of underprivileged high school seniors, a random sample of participants in the program will be selected and their mathematics scores will be compared with the previous year’s statewide average of 525 for underprivileged seniors. The researchers want to determine whether the experimental program has increased the mean achievement level over the previous year’s statewide average.

(a) If \( \alpha = 0.05 \), what sample size is needed to have a probability of Type II error of at most 0.025 if the actual mean is increased to 550? From previous results, \( \sigma \approx 80 \).

Answer: 
\[
n = \frac{\sigma^2 \cdot (Z_\alpha + Z_\beta)}{\Delta^2} = \frac{80^2 \cdot (1.645 + 1.96)^2}{(525 - 550)^2} = 133.1 \Rightarrow n = 134
\]

(b) Suppose a random sample of 100 students is selected yielding \( \bar{x} = 542 \) and \( s = 76 \). Is there sufficient evidence to conclude that the mean mathematics achievement level has been increased? Find the p-value of the test.

Answer: Large sample one population mean Z test.

Hypothesis: \( H_0 : \mu = 525, \quad H_\alpha : \mu > 525 \)

\[
Z_\alpha = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{542 - 525}{\frac{76}{\sqrt{100}}} = 2.24
\]

Since \( Z_\alpha > Z_{0.05} = 1.645 \), we reject \( H_0 \) which means there is sufficient evidence to conclude that the mean mathematics achievement level has been increased.
\[ p - value = P(Z > Z_o) = P(Z > 2.24) = 0.0125. \]

3. A federal regulatory agency is investigating an advertised claim that a certain device can increase the gasoline mileage of cars (mpg). Ten such devices are purchased and installed in cars belonging to the agency. Gasoline mileage for each of the cars is recorded both before and after installation. The data are recorded here.

<table>
<thead>
<tr>
<th>cars</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Before (mpg)</td>
<td>19.1</td>
<td>29.9</td>
<td>17.6</td>
<td>20.2</td>
<td>23.5</td>
<td>26.8</td>
<td>21.7</td>
<td>25.7</td>
<td>19.5</td>
<td>28.2</td>
</tr>
<tr>
<td>After (mpg)</td>
<td>25.8</td>
<td>23.7</td>
<td>28.7</td>
<td>25.4</td>
<td>32.8</td>
<td>19.2</td>
<td>29.6</td>
<td>22.3</td>
<td>25.7</td>
<td>20.1</td>
</tr>
<tr>
<td>Change (mpg)</td>
<td>6.7</td>
<td>-6.2</td>
<td>11.1</td>
<td>5.2</td>
<td>9.3</td>
<td>-7.6</td>
<td>7.9</td>
<td>-3.4</td>
<td>6.2</td>
<td>-8.1</td>
</tr>
</tbody>
</table>

(a) Place 90% confidence intervals on the average mpg for both the before and after phases of the study. Interpret these intervals. Does it appear that the device will significantly increase the average mileage of cars?

**Answer:**

Before: \( \bar{x} = 23.22, \ s = 4.25; \)

After: \( \bar{x} = 25.33, \ s = 4.25; \)

Change: \( \bar{d} = 2.11, \ s_d = 7.54; \)

Before: \( 23.22 \pm 1.833 \cdot 4.25/\sqrt{10} \Rightarrow (20.76, 25.68) \)

After: \( 25.33 \pm 1.833 \cdot 4.25/\sqrt{10} \Rightarrow (22.87, 27.79) \)

Since the two interval overlaps, there is not strong evidence of an increase in the average mpg after installing the device.

(b) Test at \( \alpha = 0.05 \), please report the p-value.

**Answer:** Small sample two population mean paired t test.

Hypothesis: \( H_o: \mu_{after} - \mu_{before} = 0, \ H_a: \mu_{after} - \mu_{before} > 0 \)

\[ t_o = \frac{\bar{d} - 0}{s_d/\sqrt{n}} = \frac{2.11 - 0}{7.54/\sqrt{10}} = 0.88 \]

Since \( t_o < t_{0.05, df=9} = 1.833 \), we fail to reject \( H_o \) and conclude there is not strong evidence of an increase in the average mpg after installing the device.

\[ p - value = P(t > t_o) = P(t > 0.88) = 0.2009 > 0.05. \ (\text{or since} \ 0.703 < 0.88 < 1.383, 0.10 < p - value < 0.25) \]