5.4.3
(a) Z is a standard normal random variable with known variance, thus we simply treat -2.81 and 2.75 as coming from a standard normal distribution.
\[ P(-2.81 \leq Z \leq 2.75) = k = 0.9945 \]
\[ P\left(\bar{x} - 2.81 \frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{x} + 2.75 \frac{\sigma}{\sqrt{n}}\right) = 0.9945 \]
(b)
(c) 0.9945
\[ P\left(\bar{x} - 1.96 \frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{x} + 1.96 \frac{\sigma}{\sqrt{n}}\right) = 0.95 \]
(d)

5.4.4
\[ P(15.65 - 1.96 \frac{0.59}{\sqrt{50}} \leq \mu \leq 15.65 + 1.96 \frac{0.59}{\sqrt{50}}) = P(15.4865 \leq \mu \leq 15.8135) = 0.95 \]

5.4.9

5.5.1
a)
\[ \hat{p} = .35 \quad , \quad n = 1200 \quad , \quad \alpha = .05 \]
\[ \hat{p} \pm z_{\alpha} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = .35 \pm 1.96 \sqrt{.35(.65)/1200} \]
95\% Confidence Interval:
(0.323 , 0.377)
b)
\[ \hat{p} = .6 \quad , \quad n = 1200 \quad , \quad \alpha = .05 \]
\[ \hat{p} \pm z_{\alpha} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = .6 \pm 1.96 \sqrt{.6(.4)/1200} \]
95\% Confidence Interval:
(0.572 , 0.628)
c)
\[ \hat{p} = .15, \quad n = 1200, \quad \alpha = .05 \]
\[ \hat{p} \pm z_\alpha \sqrt{\hat{p}(1 - \hat{p})/n} = .15 \pm 1.96\sqrt{.15(.85)/1200} \]

95% Confidence Interval:
(.1498, .1502)

d)

For all 3 cases, we assume that \( \hat{P} \sim \text{Normal} \left( p, \frac{p(1 - p)}{n} \right) \).

For case (a), we are 95% confident that the true proportion of people who find political advertising to be untrue lie between (.323, .377).

For case (b), we are 95% confident that the true proportion of voters who will not vote for candidates whose advertisements are considered to be untrue lie between (.572, .628).

For case (c), we are 95% confident that the true proportion of those who avoid voting for candidates whose advertisements are considered untrue and who have complained to the media or to the candidate about the falsehood in commercials lie between (.323, .377).

5.5.2

We assume that \( \bar{X} \sim \text{Normal} \left( \mu, \frac{\sigma^2}{n} \right) \).

\[ \bar{x} = 12.12, \quad s = 4.7, \quad n=49, \quad \alpha=.02 \]
\[ \bar{x} \pm z_\alpha \left( \frac{s}{\sqrt{n}} \right) = 12.12 \pm 2.33 \left( \frac{4.7}{7} \right) \]

98% Confidence Interval:
(10.556, 13.684)

We are 98% confident that the true mean for P/E multiples lie between (10.556, 13.684).

5.5.3

a)
\[ L(\mu, \sigma) = \prod_{i=1}^{n} \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x_i - \mu)^2}{2\sigma^2}} = \sigma^{-n} \left(2\pi\right)^{-\frac{n}{2}} e^{-\frac{\sum_{i=1}^{n} (x_i - \mu)^2}{2\sigma^2}} \]

\[ \ln(L) = -n \ln(\sigma) - \frac{n}{2} \ln(2\pi) - \sum_{i=1}^{n} \frac{(x_i - \mu)^2}{2\sigma^2} \]

\[ \frac{\delta(\ln(L))}{\delta \mu} = -\sum_{i=1}^{n} \frac{(x_i - \mu)}{\sigma^2} = 0 \]

\[ \hat{\mu}_{MLE} = \frac{\sum_{i=1}^{n} x_i}{n} = \bar{x} \]

Therefore, \( \hat{\mu} = \bar{x} \)

b) \[ P \left( -\frac{2\sigma}{\sqrt{n}} < \mu < \frac{2\sigma}{\sqrt{n}} \right) = P \left( -\frac{2\sigma}{\sqrt{n}} < -\bar{x} + \mu < \frac{2\sigma}{\sqrt{n}} \right) = P \left( \frac{2\sigma}{\sqrt{n}} > \bar{x} - \mu > -\frac{2\sigma}{\sqrt{n}} \right) \]

\[ = P \left( -\frac{2\sigma}{\sqrt{n}} < \bar{x} - \mu < \frac{2\sigma}{\sqrt{n}} \right) = P \left( -2 < \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} < 2 \right) = P(-2 < Z < 2) = 1 - 2P(Z > 2) = 0.954 \]

c) \[ P \left( -\frac{k\sigma}{\sqrt{n}} < \mu < \frac{k\sigma}{\sqrt{n}} \right) = P \left( -\frac{k\sigma}{\sqrt{n}} < -\bar{x} + \mu < \frac{k\sigma}{\sqrt{n}} \right) = P \left( \frac{k\sigma}{\sqrt{n}} > \bar{x} - \mu > -\frac{k\sigma}{\sqrt{n}} \right) \]

\[ = P \left( -\frac{k\sigma}{\sqrt{n}} < \bar{x} - \mu < \frac{k\sigma}{\sqrt{n}} \right) = P \left( -k < \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} < k \right) = P(-k < Z < k) = 1 - 2P(Z > k) = 0.90 \Rightarrow P(Z > k) = 0.05 \Rightarrow k = 1.645 \]

5.5.4

We assume that \( \bar{X} \sim \text{Normal} \left( \mu, \frac{110}{45} \right) \).

\( \bar{x} = 68.51 \), \( \sigma = \sqrt{\frac{110}{45}} \), \( n=45 \), \( \alpha=0.05 \)

\[ \bar{x} \pm z_{\frac{\alpha}{2}} \left( \frac{\sigma}{\sqrt{n}} \right) = 68.51 \pm 1.96 \left( \frac{\sqrt{\frac{110}{45}}}{\sqrt{45}} \right) \]

95% Confidence Interval:

(65.45, 71.57)

We are 95% confident that the true mean lie between (65.45, 71.57).

5.5.5

We assume that \( \hat{p} \sim \text{Normal} \left( p, p(1-p)/n \right) \)
\[
\hat{p} = \frac{18}{50} = .36 \quad n = 50 \quad \alpha = .02
\]
\[
\hat{p} \pm z_{\alpha/2} \sqrt{\hat{p}(1-\hat{p})/n} = .36 \pm 2.33 \sqrt{.36(.64)/50}
\]
98% Confidence Interval:
(.202, .518)

We are 98% confident that the true proportion of seniors planning to pursue a graduate degree lies between (.202, .518).

5.5.6

We assume that \( \hat{p} \sim Normal \left( p, p(1-p)/n \right) \)

\[
\hat{p} = \frac{17}{105} = .162 \quad n = 105 \quad \alpha = .1
\]
\[
\hat{p} \pm z_{\alpha} \sqrt{\hat{p}(1-\hat{p})/n} = .162 \pm 1.645 \sqrt{.162(.838)/105}
\]
90% Confidence Interval:
(.103, .221)

We are 90% confident that the true proportion of defective DVD players lie between (.103, .221).

5.5.11

a)
\[
\hat{p} = \frac{40}{500} = .08 \quad n = 500 \quad \alpha = .1
\]
\[
\hat{p} \pm z_{\alpha} \sqrt{\hat{p}(1-\hat{p})/n} = .08 \pm 1.645 \sqrt{.08(.92)/500}
\]
90% Confidence Interval:
(.06, .1)

We are 90% confident that the true proportion of defective items lie between (.06, .1).

b)
The assumption of normality is valid, based on the sampling distribution of \( \hat{p} \) and the central limit theorem.
We are 98% confident that the true proportion of support for the president lies between (.394, .506).

5.5.22

a)


\[ z_{\alpha/2} \left( \frac{\sqrt{25}}{\sqrt{n}} \right) = E \]

\[ 1.96 \left( \frac{\sqrt{25}}{\sqrt{n}} \right) = .01 \]

\[ n = \frac{.25}{(.01 \times 1.96)^2} \]

\[ n = 650.77 \]

\[ n \geq 651 \]

b)

\[ z_{\alpha/2} \left( \frac{\sqrt{25 \times (.75)}}{\sqrt{n}} \right) = E \]

\[ 1.96 \left( \frac{\sqrt{25 \times (.75)}}{\sqrt{n}} \right) = .01 \]

\[ n = \frac{.25 \times (.75)}{(0.01 \times 1.96)^2} \]

\[ n = 488.08 \]

\[ n \geq 489 \]

5.5.30

\[ \bar{x} = 5.7 \quad , \quad s = .19 \quad , \quad n=20 \quad , \quad \alpha=.02 \]

\[ \bar{x} \pm t_{\alpha} \left( \frac{s}{\sqrt{n}} \right) = 5.7 \pm 2.39 \left( \frac{.19}{\sqrt{20}} \right) \]

98% Confidence Interval:

(3.32, 8.08)

We are 98% confident that the mean change in heart rate lie between (3.32, 8.08).

5.5.31

\[ \bar{x} = .905 \quad , \quad s = .005 \quad , \quad n=10 \quad , \quad \alpha=.05 \]

\[ \bar{x} \pm t_{\alpha} \left( \frac{s}{\sqrt{n}} \right) = .905 \pm 2.26 \left( \frac{.005}{\sqrt{10}} \right) \]

95% Confidence Interval:

(.9014, .9086)

We are 95% confident that the mean diameter of bearings made lie between (.9014, .9086).
5.5.32
\[ \bar{x} = 61.22 \quad , \quad s = 3.32 \quad , \quad n = 10 \quad , \quad \alpha = 0.05 \]
\[ \bar{x} \pm t_{\alpha} \frac{s}{\sqrt{n}} = 61.22 \pm 2.262 \left( \frac{3.32}{\sqrt{10}} \right) \]
95% Confidence Interval:
(58.85, 63.59)
We are 95% confident that the actual average air pollution index for this city lie between (58.85, 63.59).

5.5.41
a)

Looking at the normal plot, the points are mostly around the normal line suggesting that the data comes from a normal population.

b)
\[ \bar{x} = 148.18 \quad , \quad s = 1.38 \quad , \quad n = 10 \quad , \quad \alpha = 0.05 \]
\[ \bar{x} \pm t_{\alpha} \frac{s}{\sqrt{n}} = 148.18 \pm 2.262 \left( \frac{1.38}{\sqrt{10}} \right) \]
95% Confidence Interval:
(147.193, 149.167)
We are 95% confident that the population mean stopping distance \( \mu \) lie between (147.193, 149.167).