This is a close book exam. Please turn in by 11:20am, the end of this lecture. Please include detailed solutions for full credit.

1. The gunner on a small assault boat fires three missiles at an attacking plane. Each has a 20% chance of being on target. If two or more of the shells find their mark, the plane will crash. At the same time, the pilot of the plane fires 4 air-to-surface rockets, each of which has a 0.1 chance of destroying the boat. If one or more of the rockets hit the boat, the boat will sink. Would you rather be on the plane or the boat? (That is, please calculate and compare the probability that the plane will crash and the probability that the boat will be destroyed.)

**Solution:** Let $X$ denotes the # of shells find their mark. Then $X \sim B (3, 0.2)$. Let $Y$ denotes the # of rockets destroy the boat. Then $Y \sim B (4, 0.1)$.

\[
P(\text{Plane crashes}) = P(X = 2) + P(X = 3) \\
= \binom{3}{2} (0.2)^2 (1 - 0.2) + \binom{3}{3} (0.2)^3 = 0.104
\]
\[
P(\text{Boat destroyed}) = P(Y \geq 1) \\
= 1 - P(Y = 0) \\
= 1 - \binom{4}{0} (1 - 0.1)^4 \\
= 0.3439
\]
Therefore, on the plane will be better.

2. Let $X \sim N(\mu_1, \sigma_1^2)$, and $Y \sim N(\mu_2, \sigma_2^2)$. Furthermore, $X$ and $Y$ are independent to each other. Please derive the distribution of $2X-3Y+5$

**Solution:** We have
\[
M_X(t) = \exp \left( \mu_1 t + \frac{1}{2} \sigma_1^2 t^2 \right) \\
M_Y(t) = \exp \left( \mu_2 t + \frac{1}{2} \sigma_2^2 t^2 \right)
\]
Since $X$ and $Y$ are independent, we have
\[
M_{2X-3Y+5}(t) = E(e^{t(2X-3Y+5)}) \\
= E(e^{2tX-3tY+5t}) = E(e^{2tX} e^{-3tY} e^{5t}) \\
= E(e^{2tX}) E(e^{-3tY}) E(e^{5t}) \\
= M_X(2t) M_Y(-3t) \cdot e^{5t} \\
= \exp \left( (2\mu_1 - 3\mu_2 + 5) t + \frac{1}{2} (4\sigma_1^2 + 9\sigma_2^2) t^2 \right)
\]
\[
\Rightarrow 2X - 3Y + 5 \sim N(2\mu_1 - 3\mu_2 + 5, 4\sigma_1^2 + 9\sigma_2^2)
\]