1. Let $Z_t \sim \text{i.i.d.}(0, \sigma^2)$, that is, an independent and identically distributed sequence of white noise with mean 0 and constant variance. Please show that the AR(1) series

$$X_t = \beta_0 + \beta_1 X_{t-1} + Z_t,$$

is stationary iff $|\beta_1|<1$.

2. Let $Z_t \sim \text{i.i.d.}(0, \sigma^2)$, that is, an independent and identically distributed sequence of white noise with mean 0 and constant variance. Please show that the MA(1) process:

$$X_t = \beta_1 Z_{t-1} + Z_t,$$

is invertible iff $|\beta_1|<1$.

3. For the time series $X_t = 4 + Z_t + 0.6Z_{t-1}$ where $\{Z_t\}$ is a series of white noise with mean 0 and variance 1. Please derive:
   (a). Is this series stationary?
   (b). Is this series invertible?
   (c). The mean of the series.
   (d). The variance of the series.
   (e). The autocovariance functions and the autocorrelation functions of the series.

4. For the AR(2) process $2X_t = X_{t-1} + 0.5X_{t-2} + Z_t$, where $\{Z_t\}$ is a series of white noise with mean 0 and variance $\sigma^2$. Please derive:
   (a). Is this series stationary?
   (b). Is this series invertible?
   (c). The mean of the series.
   (d). The variance of the series.
   (e). The autocovariance functions and the autocorrelation functions of the series.

5. Let $Z_t \sim \text{i.i.d.}(0, \sigma^2)$, that is, an independent and identically distributed sequence of white noise with mean 0 and constant variance. Please derive the Yule-Walker equation for a stationary AR(2) time series: $X_t = \beta_0 + \beta_1 X_{t-1} + \beta_2 X_{t-2} + Z_t$

6. Consider the stationary and invertible ARMA(1,1) process $X_t = \alpha X_{t-1} + Z_t + \beta Z_{t-1}$, where $\{Z_t\}$ is a series of white noise with mean 0 and variance $\sigma^2$.
   (a) For what values of $\alpha$ and $\beta$ is this process stationary, and invertible? Please show detailed derivations.
   (b) Please derive its mean, variance, auto-covariances and auto-correlations.