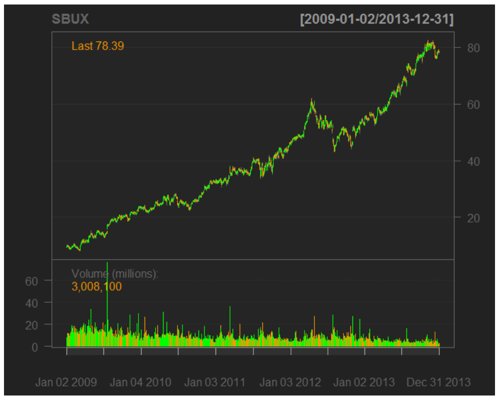
AMS586   Si Wen

Report for Fitting a Time Series Model

1. Introduction

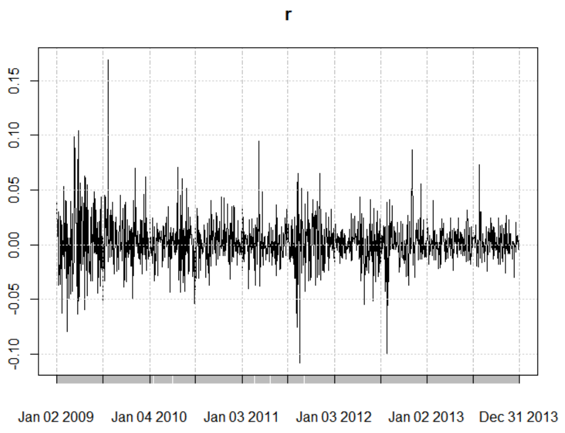
In this report, I choose the stock price of Starbucks Corporation (SBUX) as my data to analyze. The time period for the data is from Jan 2009 to Dec 2013. The total number of data points is 1258. The line chart for the close price during that period is shown as below.



The structure of this report is as follows: in the second section, I will illustrate the method of modeling, which including the discussion about trend and seasonality and fitting the models for the random component; then, it will be followed by the diagnostic and comparison of the models in the third section; in the last section, there will be a short conclusion.

2. Method of modeling

To analyze the stock price, we usually calculate the logged return of the stock to make the data stationary. The following plot shows the daily logged return of SBUX.



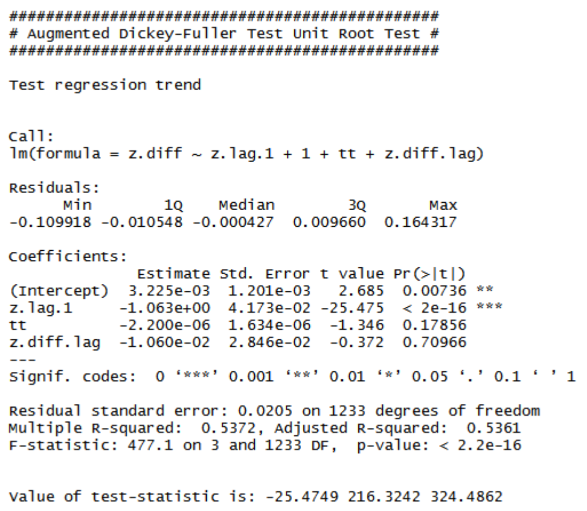
From the above plot, it seems that there is no trend or seasonality for this time series and the data has mean 0.

a. Trend

To test whether there is a drift or a trend for the return, I used the Augmented Dickey–Fuller (ADF) test. The model for the ADF test is

$$\Delta X_t = \alpha + \beta t + \gamma X_{t-1} + \delta_1 \Delta X_{t-1} + \cdots + \delta_{p-1} \Delta X_{t-p-1} + \varepsilon_t$$

Imposing the constraints $\alpha = 0$ and $\beta = 0$ corresponds to modelling a random walk without a drift, and using the constraint $\beta = 0$ corresponds to modelling a random walk with a drift. The lag p of the difference can be determined by AIC of the fitted AR models. The result of the test is as following.



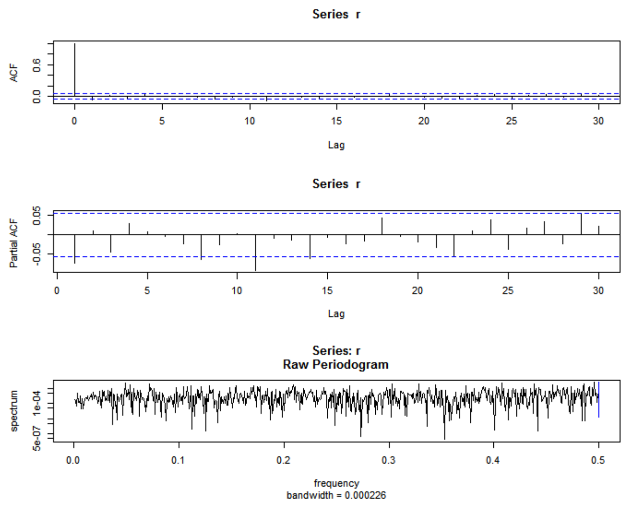
```
###############################################
# Augmented Dickey-Fuller Test Unit Root Test #
###############################################

Test regression trend


Call:
lm(formula = z.diff ~ z.lag.1 + 1 + tt + z.diff.lag)

Residuals:
      Min        1Q    Median        3Q       Max
-0.109918 -0.010548 -0.000427  0.009660  0.164317

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  3.225e-03  1.201e-03   2.685  0.00736 **
z.lag.1     -1.063e+00  4.173e-02 -25.475  < 2e-16 ***
tt          -2.200e-06  1.634e-06  -1.346  0.17856
z.diff.lag  -1.060e-02  2.846e-02  -0.372  0.70966
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.0205 on 1233 degrees of freedom
Multiple R-squared:  0.5372, Adjusted R-squared:  0.5361
F-statistic: 477.1 on 3 and 1233 DF,  p-value: < 2.2e-16


Value of test-statistic is: -25.4749 216.3242 324.4862
```

From the result, we can see that the intercept, which is α, is significantly different from 0. It means that the mean of the time series is not 0, in other words, there is a drift. Also, there is no linear trend for this time series, since the coefficient for tt is not significant.

Additionally, this test also indicates there is no unit root present since the null hypothesis of $\gamma = 0$ is rejected. This means the model is not a random walk. We also notice that the $\hat{\gamma} \approx -1.063$ which means the AR part of the model is stationary.

b. Seasonality

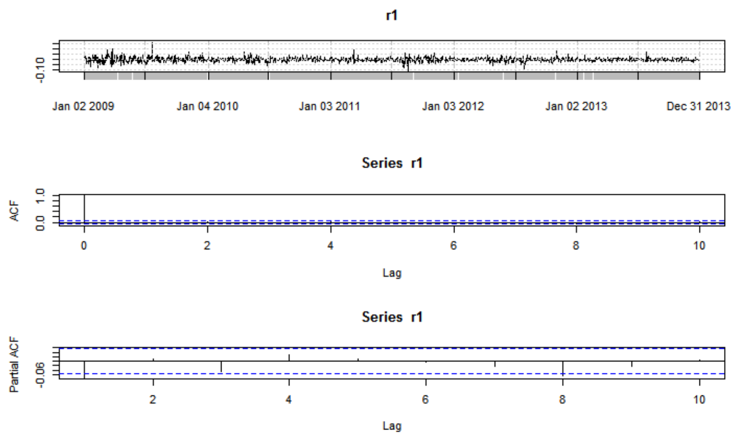
By viewing the ACF, PACF, and spectrum Periodgram, we cannot find an evidence for seasonality.



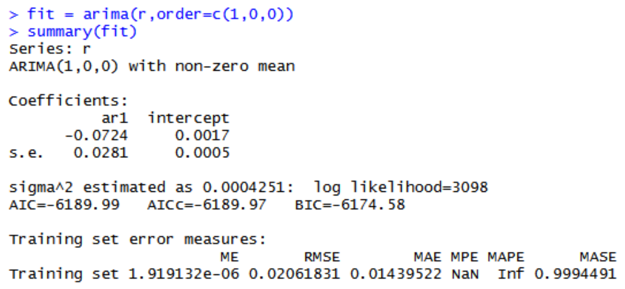
c. Random component

To remove the drift from the time series, we can use the two different methods: demeaning the data and making the difference.

First, I try to demean the data and fit the demeaned data with a time series model. After deducting the mean of the series, the time plot, ACF and PACF are as following.

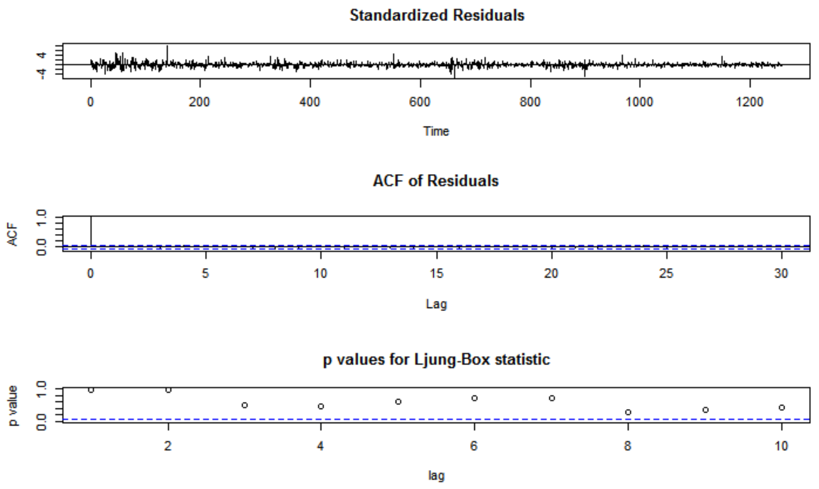


Since the Partial ACF cut off after the first lag, it seems that the demeaned logged return follows AR(1) model.

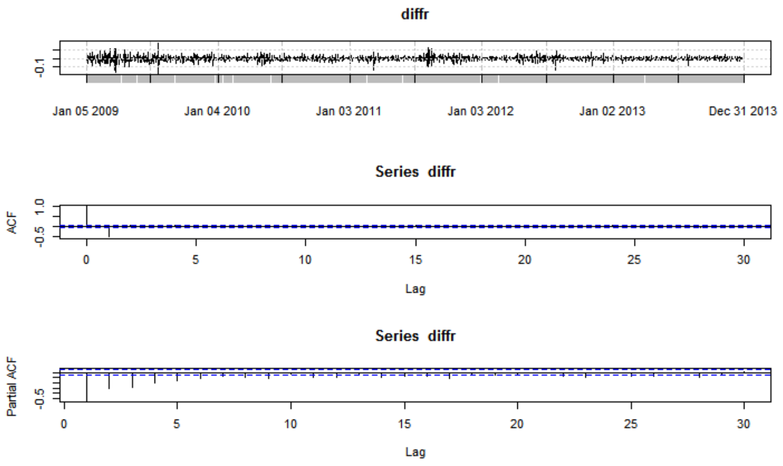


```
> fit = arima(r,order=c(1,0,0))
> summary(fit)
Series: r
ARIMA(1,0,0) with non-zero mean

Coefficients:
          ar1  intercept
      -0.0724     0.0017
s.e.   0.0281     0.0005

sigma^2 estimated as 0.0004251:  log likelihood=3098
AIC=-6189.99   AICc=-6189.97   BIC=-6174.58

Training set error measures:
                       ME       RMSE        MAE MPE MAPE      MASE
Training set 1.919132e-06 0.02061831 0.01439522 NaN  Inf 0.9994491
```

The diagnostic of residuals of the AR(1) model with drift is summarized in the following. From the ACF plot and the Ljung-Box statistics, we can see that the residuals are almost uncorrelated.



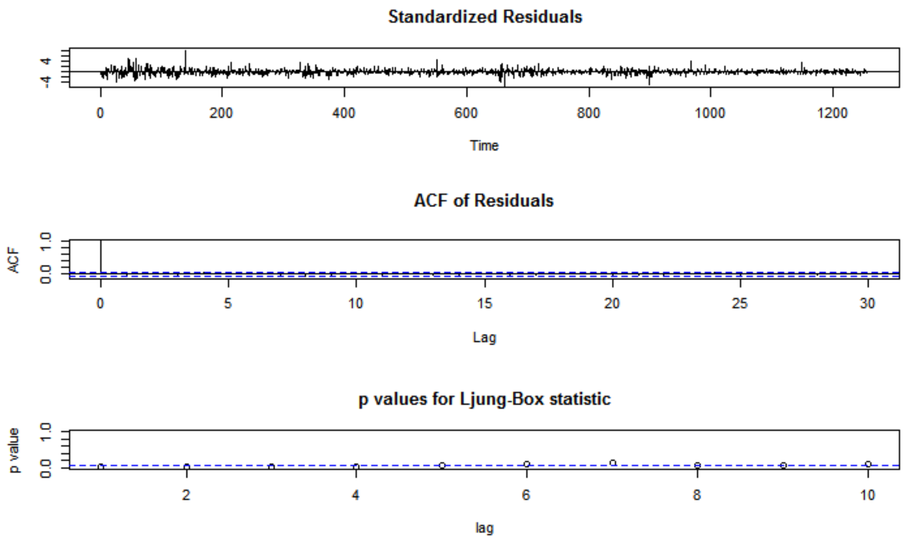
We can also get rid of the drift by making first difference. Then the time plot, ACF and PACF are listed below.



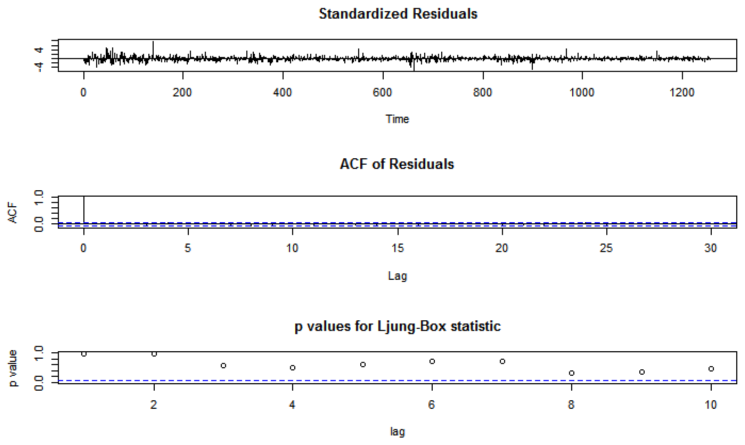
Since the ACF cut off after the first lag and the Partial ACF decrease gradually, it seems that the differenced logged return follows MA(1) model.

```
> fit1 = arima(r, order=c(0,1,1))
> summary(fit1)
Series: r
ARIMA(0,1,1)

Coefficients:
          ma1
      -1.0000
s.e.   0.0042

sigma^2 estimated as 0.0004277:  log likelihood=3088.16
AIC=-6172.33   AICc=-6172.32   BIC=-6162.06

Training set error measures:
                        ME       RMSE        MAE MPE MAPE     MASE
Training set -0.0007476267 0.02067255 0.01441089 NaN  Inf 1.000537
```

The residual seems to be not independent from each other. To solve this, I tried ARIMA(1,1,1).



```
> fit2 = arima(r, order=c(1,1,1))
> summary(fit2)
Series: r
ARIMA(1,1,1)

Coefficients:
          ar1      ma1
      -0.0720  -0.9995
s.e.   0.0282   0.0050

sigma^2 estimated as 0.0004256:  log likelihood=3091.4
AIC=-6176.79   AICc=-6176.77   BIC=-6161.38

Training set error measures:
                        ME       RMSE        MAE MPE MAPE      MASE
Training set -0.0007747377 0.02062276 0.01439253 NaN  Inf 0.9992619
```



The residuals for ARIMA(1,1,1) are much better than ARIMA(0,1,1).

3. Model selection and diagnostic

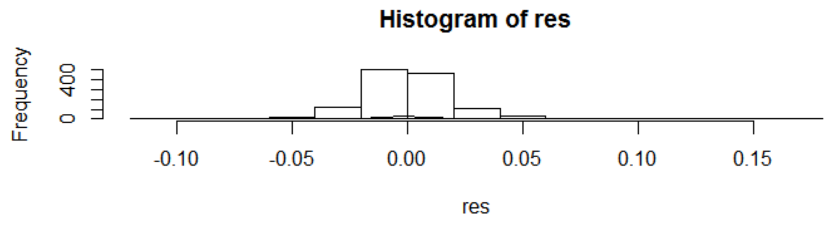
To compare the two models, I make a comparison of the information criteria. From the following table, we can see that the AR(1) model with intercept is much better than that of ARIMA(1,1,1).

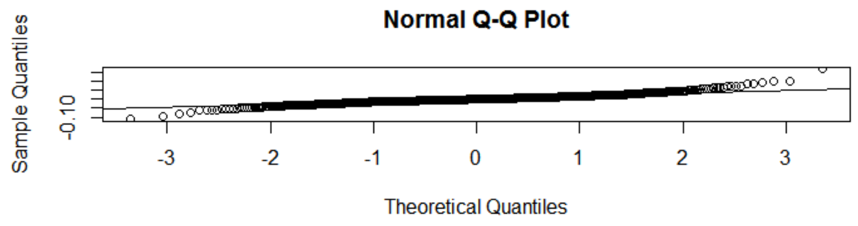
model	Log-likelihood	AIC	AICC	BIC
AR(1)+intercept	3098	-6189.99	-6189.97	-6174.58
ARIMA(1,1,1)	3091.4	-6176.79	-6176.77	-6161.38

Thus the final model is

$$X_t = 0.0017 - 0.0724X_{t-1} + \varepsilon_t$$

The diagnostic for the independence of the residual has been shown in the previous section. Besides that, we can also check the normality of the residuals. From the histogram and qq-plot of the residual, we can see that the residuals are not normal distirbuted. The flat density curve and the invert S-shaped qq-plot indicate that the denisty of the residual should be fat tailed.





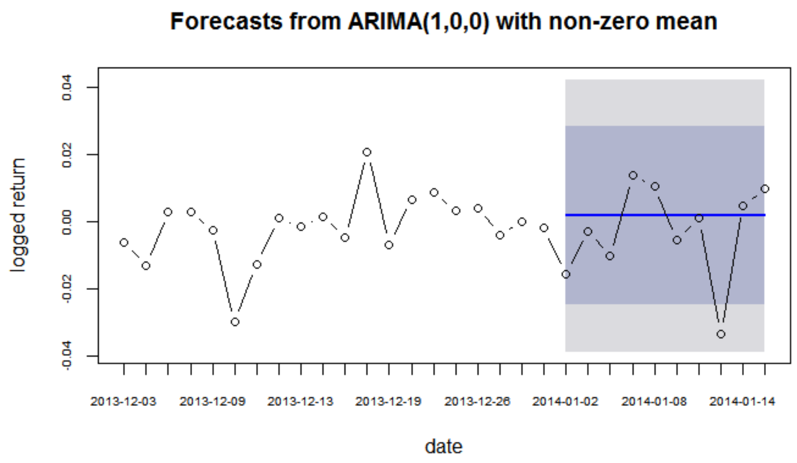
4. **Forecasting**

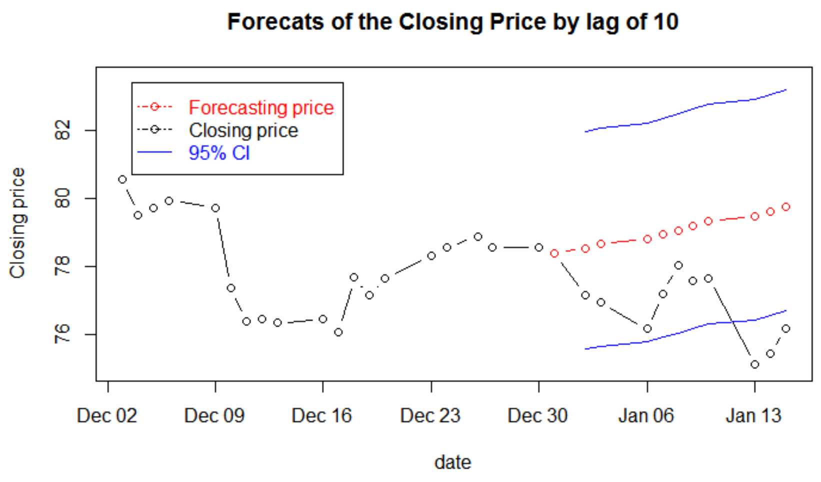
Using the AR(1) model with drift:

$$X_t = 0.0017 - 0.0724X_{t-1} + \varepsilon_t$$

to forecast the logged return, I get the following result.

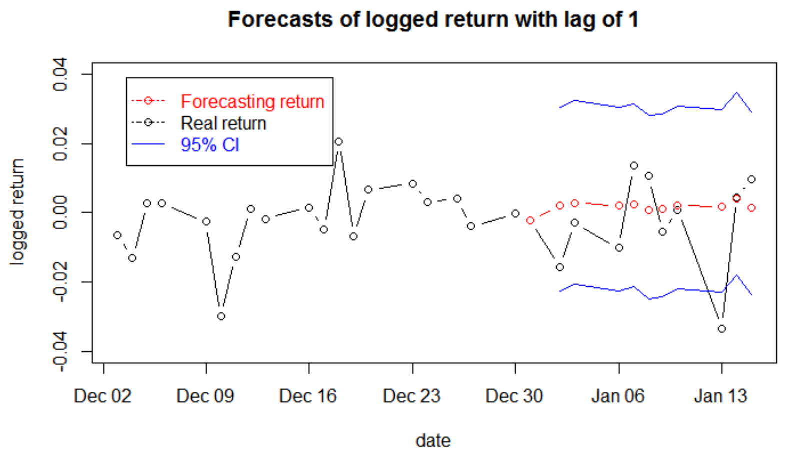
```
Forecasts:
     Point Forecast       Lo 80      Hi 80       Lo 95      Hi 95
1259    0.001948632 -0.02447479 0.02837206 -0.03846251 0.04235978
1260    0.001659766 -0.02483290 0.02815243 -0.03885727 0.04217680
1261    0.001680691 -0.02481234 0.02817372 -0.03883690 0.04219828
1262    0.001679175 -0.02481386 0.02817221 -0.03883842 0.04219677
1263    0.001679285 -0.02481375 0.02817232 -0.03883831 0.04219688
1264    0.001679277 -0.02481375 0.02817231 -0.03883832 0.04219687
1265    0.001679278 -0.02481375 0.02817231 -0.03883832 0.04219687
1266    0.001679278 -0.02481375 0.02817231 -0.03883832 0.04219687
1267    0.001679278 -0.02481375 0.02817231 -0.03883832 0.04219687
1268    0.001679278 -0.02481375 0.02817231 -0.03883832 0.04219687
```

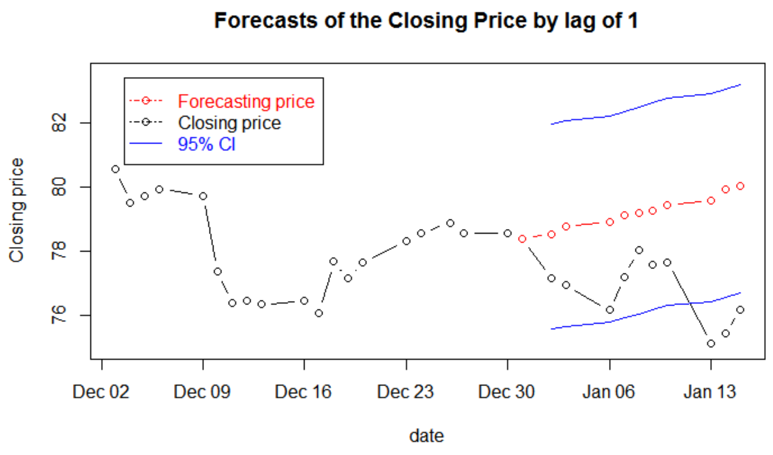




From the results and the figures above, we can see that the forecasts tend to be the same after several steps. This can be explained by the ACF plot. Since the autocorrelation is quite small after the 1st lag, the h-step-ahead forecast is not reliable. So I tried to do 1-step-ahead forecast, then re-fit the time series with newly added observations and then predict the next one.

```
              Forecasts        H80        H95         L80         L95
2014-01-02 0.0019486322 0.02837206 0.04235978 -0.02447479 -0.03846251
2014-01-03 0.0029209698 0.02934157 0.04332779 -0.02349963 -0.03748585
2014-01-06 0.0019877603 0.02839870 0.04237980 -0.02442318 -0.03840428
2014-01-07 0.0025058222 0.02890994 0.04288744 -0.02389830 -0.03787579
2014-01-08 0.0008010308 0.02719769 0.04117124 -0.02559563 -0.03956918
2014-01-09 0.0010272904 0.02741585 0.04138511 -0.02536127 -0.03933053
2014-01-10 0.0021823161 0.02856149 0.04252578 -0.02419686 -0.03816115
2014-01-13 0.0017174421 0.02808623 0.04204502 -0.02465134 -0.03861014
2014-01-14 0.0041615849 0.03055023 0.04451954 -0.02222706 -0.03619637
2014-01-15 0.0014293794 0.02780761 0.04177141 -0.02494885 -0.03891265
```





From the plots, it seems that the forecasts of the logged return of lag 1are more close to the real data. However, by comparing the sum squared error, we can get the opposite conclusion. This can be explained by the small sample size we are using, only 10 forecast values. If we do testing based on larger sample size, the forecast result for the 1-step-ahead forecasting should be much better.

```
> sum((fore.mean-as.vector(ret[period[1+1:l]]))^2)#SSE of lag10
[1] 0.002045399
> sum((fore2.mean2-as.vector(ret[period[1+1:l]]))^2)#SSE of lag1
[1] 0.002054035
```

5. Conclusion

I fit the logged return of SBUX data in a AR(1) model with drift:

$$X_t = 0.0017 - 0.0724X_{t-1} + \varepsilon_t$$

The ε_t are independent but not normally distributed. Its density must be fat tail.

R code:

```
require(quantmod)
require(forecast)
require(urca)
require(tseries)

##load the data
getSymbols('SBUX')
chartSeries(SBUX,subset='2009::2013')

##Calculate the log-return
ret = na.omit(diff(log(SBUX$SBUX.Close)))
r = ret["2009::2013"]
n=length(r)
plot(r)

##Check for the trend
summary(ur.df(r, type='trend', lags=20, selectlags="BIC"))

##Check for the seasonality
par(mfrow=c(3,1))
acf(r)
pacf(r)
spec.pgram(r)

##1.Demean
r1=r-mean(r)
par(mfrow=c(3,1))
plot(r1)
acf(r1,lag=10)
pacf(r1,lag=10)

fit = arima(r,order=c(1,0,0))
summary(fit)
tsdiag(fit)

##2.Difference
diffr = na.omit(diff(r))

par(mfrow=c(3,1))
plot(diffr)
```

```
acf(diffr)
pacf(diffr)

##Fit the ARIMA model
fit1 = arima(r, order=c(0,1,1))
summary(fit1)
tsdiag(fit1)

fit2 = arima(r, order=c(1,1,1))
summary(fit2)
tsdiag(fit2)

##Check by auto.arima
auto.arima(r)

##Diagnostic
res=residuals(fit)
shapiro.test(res)

par(mfrow=c(2,1))
hist(res)
lines(density(res))
qqnorm(res)
qqline(res)

########################################################
##  HW4 Forcasting
########################################################
dev.off()

##Forecasting of lag 10
l=10  # number of lags for forecasting
h=20  # number of training data shown in the plot

fore <- forecast(fit,l)
summary(fore)
########################################################

##Plot the forecasting logged return and the real value
plot(fore,h,axes=FALSE,ylab="logged return",xlab="date",type="b")
lines(c(n+0:l),ret["2013-12-31::"][1+0:l],type="b")
```

```
#combine the time period of last h terms in training data and the
testing data of length l
date=c(index(r[n-0:(h-1)]),index(ret["2014-01-01::"][1:l]))

#add x-axis and y-axis
axis(1, at = c(n-h+1:(h+l)), labels = date, cex.axis=0.6)
axis(2,cex.axis=0.6)
box()
######################################################

##Calculating the Forecasts of closing price
fore.mean <- as.vector(fore$mean) #Change the estimated mean to
a vector
#change the last closing price in the training data to a number
lastprice <- as.numeric(SBUX$SBUX.Close["2013-12-31"])

fore.price    <-    Reduce(function(x,y)    {x*exp(y)},    fore.mean,
init=lastprice, accumulate=T)

#95% Upper and Lower bond for closing price
lower=fore.price[c(1+1:l)]*exp(fore$lower[,2])
upper=fore.price[c(1+1:l)]*exp(fore$upper[,2])
######################################################

##Plot the forecasting closing price and the real value
plot(date,SBUX$SBUX.Close[date],type="b",ylab="Closing
price",ylim=c(75,83.5),
   main="Forecats of the Closing Price by lag of 10")
period=index(ret["2013-12-31::"][1+0:l]) #the forecast period
lines(period,fore.price,type="b",col="red")
lines(period[1+1:l],upper,col="blue")
lines(period[1+1:l],lower,col="blue")
legend("topleft",  c("Forecasting  price","Closing  price","95%  CI"),
col=c("red","black","blue"),
    text.col=c("red","black","blue"),lty=c(4,4,1),        pch        =
c(1,1,NA),inset = .05)
######################################################

##Forecasting with lag of 1
r2=c(r,ret["2014-01-01::"][1:l])
fore2.mean=ret["2013-12-31::"][1+0:l]
fore2.upper=vector()
```

```
fore2.lower=vector()
## Loop to overlay early forecasts
for (j in seq(0, l-1, by=1)) {

   b.fit  <-auto.arima(r2[1:(n+j)])
   b.pred <- forecast(b.fit, 1)
   fore2.mean[j+2]=b.pred$mean
   fore2.upper=rbind(fore2.upper,b.pred$upper)
   fore2.lower=rbind(fore2.lower,b.pred$lower)
}
fore2 <- cbind(fore2.mean[1+1:l],fore2.upper,fore2.lower)
colnames(fore2) <- c("Forecasts","H80","H95","L80","L95")
fore2
##########################################################

##Plotting
plot(date,r2[date],type="b",ylab="logged return",
    ylim=c(-0.04,0.04),main="Forecasts of logged return with lag of
1")

lines(period,fore2.mean,type="b",col="red")
lines(period[1+1:l],fore2.mean[1+1:l]+fore2.upper[,1],col="blue")
lines(period[1+1:l],fore2.mean[1+1:l]+fore2.lower[,1],col="blue")
legend("topleft", c("Forecasting return","Real return","95% CI"),
col=c("red","black","blue"),
     text.col=c("red","black","blue"),lty=c(4,4,1),           pch           =
c(1,1,NA),inset = .05)
##########################################################

##Calculating the Forecasts of closing price
fore2.mean2=as.vector(fore2.mean[1+1:l])
fore2.price     <-     Reduce(function(x,y)     {x*exp(y)},fore2.mean2,
init=lastprice, accumulate=T)
lower2=fore2.price[c(1+1:l)]*exp(fore2.lower[,2])
upper2=fore2.price[c(1+1:l)]*exp(fore2.upper[,2])
##########################################################

##Plot the forecasting closing price and the real value
plot(date,SBUX$SBUX.Close[date],type="b",ylab="Closing
price",ylim=c(75,83.5),
    main="Forecasts of the Closing Price by lag of 1")
period=index(ret["2013-12-31::"][1+0:l]) #the forecast period
```

```
lines(period,fore2.price,type="b",col="red")
lines(period[1+1:l],upper,col="blue")
lines(period[1+1:l],lower,col="blue")
legend("topleft", c("Forecasting price","Closing price","95% CI"),
col=c("red","black","blue"),
    text.col=c("red","black","blue"),lty=c(4,4,1),            pch            =
c(1,1,NA),inset = .05)
########################################################

##Calculating the sum square error
sum((fore.mean-as.vector(ret[period[1+1:l]]))^2)#SSE of lag10
sum((fore2.mean2-as.vector(ret[period[1+1:l]]))^2)#SSE of lag1

sum((fore.price[1+1:l]-
as.vector(SBUX$SBUX.Close[period[1+1:l]]))^2)
sum((fore2.price[1+1:l]-
as.vector(SBUX$SBUX.Close[period[1+1:l]]))^2)
```