AMS 597: Statistical Computing

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Gibbs Sampler

- The Gibbs sampler is another special case of the Metropolis-Hastings sampler.
- The Gibbs sampler is often applied when the target is a multivariate distribution.
- Suppose that all the univariate conditional densities are fully specified and it is reasonably easy to sample from them.
The chain is generated by sampling from the conditional distributions, and every candidate point is accepted.

Let $X = (X_1, \ldots, X_d)$ be a random vector in $\mathbb{R}^d$.

Define the $d-1$ dimensional random vectors $X_{(-j)} = (X_1, \ldots, X_{j-1}, X_{j+1}, \ldots, X_d)$.

Denote the corresponding univariate conditional density of $X_{(j)}$ given $X_{(-j)}$ by $f(X_j | X_{(-j)})$. 

Gibbs Sampler
The Gibbs sampler generates the chain by sampling from each of the d conditional densities \( f(X_j | X_{(-j)}) \) as follows:
Gibbs Sampler

- Initialize $X(0)$ at time $t = 0$.
- For each iteration, indexed $t = 1, 2, \ldots$, repeat:
  - Set $x_1 = X_1(t - 1)$.
  - For each coordinate $j = 1, \ldots, d$
    - Generate $X^*_j(t)$ from $f(X_j | x_{(-j)})$.
    - Update $x_j = X^*_j(t)$.
  - Set $X(t) = (X^*_1(t), \ldots, X^*_d(t))$
  - Increment $t$. 
Example: Generate a bivariate normal distribution with mean vector \((\mu_1, \mu_2)\), variances \(\sigma^2_1, \sigma^2_2\), and correlation \(\rho\), using Gibbs sampling.
In general, for an arbitrary Metropolis-Hastings sampler the number of iterations that are sufficient for approximate convergence to the target distribution or what length burn-in sample is required are unknown.

A single chain may appear to have converged because the generated values have a small variance within a local part of the support set of the target distribution, but in reality the chain has not explored all of the support set.
By examining several parallel chains, slow convergence should be more evident, particularly if the initial values of the chain are overdispersed with respect to the target distribution.

You may use the Gelman-Rubin method for monitoring convergence of a M-H chain.

https://www.jstor.org/stable/2246093?seq=1#page_scan_tab_contents