AMS 310.03 Quiz 1

Fall, 1999

Instructions.
This is a closed book and closed notes exam. You can use one piece (8 x 11) of cheat sheet. Please provide detailed solutions. Anyone who cheats on the exam shall receive a score of zero.

1. Please find the mean and standard deviation of the following sample: 2, 5, 6, 3.
   Answer: $\bar{x} = 4$, $S \approx 1.826$

2. In the film “Indecent Proposal”, Demi Moore had to throw two fair dice. She would win if the sum of the two upturned faces is 7 or 11. What was her winning chance?
   Answer:
   $$P(\text{win}) = P(7) + P(11) = \frac{8}{36} = \frac{2}{9}$$

3. Columbia University has 2 major undergraduate divisions, Columbia College and the all women Barnard College. Columbia College comprises 60% of the undergraduate population and Barnard 40%. The percent of Columbia College students that commit suicide is 2%, while 1% of the Barnard girls take their own life. Taking one undergraduate student at random, what is the chance that
   (a) it is a student about to kill herself/himself?
      Answer: Let $S$, $C$, and $B$ denote the events that the student will commit suicide, is from the Columbia College, or from Barnard College respectively. Then the probability asked for here is:
      $$P(S) = P((S \cap B) \cup (S \cap C))$$
      $$= P(S \cap B) + P(S \cap C)$$
      $$= P(S \mid B)P(B) + P(S \mid C)P(C)$$
      $$= .01 \times .6 + .02 \times .6 = .016$$
   (b) it is a Barnard girl about to kill herself?
      Answer:
      $$P(S \cap B) = P(S \mid B)P(B) = .01 \times .6 = .004$$

4. There are 30 psychiatrists and 24 psychologists attending a certain conference. Three of these 54 people are randomly chosen to take part in a panel discussion. What is the probability that at least two psychiatrists are chosen?
   Answer:
   $$\binom{30}{2} \left( \frac{24}{1} \right) + \binom{30}{3} \left( \frac{24}{0} \right) \approx .585$$
5. Suppose that we have 3 cards identical in form except that both sides of the first card are colored red, both sides of the second card are colored black, and one side of the third card is colored red and the other side black. The 3 cards are mixed up in a hat, and 1 card is randomly selected and put down on the ground. If the upper side of the chosen card is colored red, what is the probability that the other side is colored black?

Answer: (Use Bayes’ Theorem) Let $RR$, $BB$, and $RB$ denote, respectively, the events that the chosen card is all red, all black, or the red-black card. Letting $R$ be the event that the upturned side of the chosen card is red, we have that the desired probability is obtained by

$$P(RB | R) = \frac{P(RB \cap R)}{P(R)} = \frac{P(R | RB) P(RB)}{P(R | RR) P(RR) + P(R | RB) P(RB) + P(R | BB) P(BB)} = \frac{(1/2)(1/3)}{(1)(1/3) + (1/2)(1/3) + (0)(1/3)} = \frac{1}{3}$$