AMS 341 (Spring, 1999)  Exam 1, March 11, 1990  Estie Arkin

1. (30 points) (a). Sketch the feasible region for the following constraints: (Make sure to label all of the constraint lines with their corresponding equation number, and shade the feasible region.)

\[ \begin{align*}
  x_1 + x_2 & \geq 1 \\
  x_1 - x_2 & \leq 2 \\
  x_2 & \leq 4 \\
  x_1 & \geq 0 \\
  x_2 & \geq 0
\end{align*} \] (1)

(b). Is the point \( x_1 = 0, x_2 = -2 \) a feasible point? Is it a basic solution?

c). Is the point \( x_1 = 1, x_2 = 1 \) a feasible point? Is it a basic solution?

d). How many feasible solutions does an LP with the above constraints have?

e). Is there an objective function for which the LP with these constraints is unbounded? If so, give such an objective function. If not, explain why.

(f). For the objective function \( \max z = -x_1 + x_2 \) the optimal solution is at point \( x_1 = 0, x_2 = 4 \). Find the range of values of \( b_3 \) (the RHS of the third constraint) for which the current basis remain optimal?

2. (25 points) A company manufactures unfinished single beds, unfinished bunk beds, and finished bunk beds. (A “bunk bed” is comprised of 2 single beds one on top of the other.) The profit on each unfinished single bed is $10, on each unfinished bunk bed $50, and on each finished bunk bed $100. Making an unfinished single bed requires 1 hour of labor. Making an unfinished bunk bed requires 2 unfinished single beds and 2 hours of labor. Making a finished bunk bed requires 1 unfinished bunk bed and 3 hours of labor. A total of 40 hours of labor are available, and there is no limit on how many beds of each type can be sold. The company wishes to maximize its profit, and asks that you formulate an LP to do so. (Your formulation does NOT have to be put into standard form. Do NOT solve, just formulate!)

(a). Define the variables you are using in the formulation.

(b). The objective function is: (min or max?)

(c). The constraints are:

3. (15 points) The following tableau is optimal for a maximization LP.

<table>
<thead>
<tr>
<th>( z )</th>
<th>( x_1 )</th>
<th>( x_2 )</th>
<th>( x_3 )</th>
<th>( x_4 )</th>
<th>RHS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>4</td>
<td>1</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>0</td>
<td>-1</td>
<td>4</td>
<td>0</td>
<td>1</td>
<td>3</td>
</tr>
</tbody>
</table>

(a). (10 points) Find another optimal BFS:

(b). (5 points) How many optimal solutions are there to this LP?

4. (10 points) A linear programming problem was solved twice using the simplex method, once with the big M method and once with the 2-phase method. Are the following possible? Explain briefly. (Each part is separate from the other part.)

(a). The big M method found a finite optimal solution in which some artificial variable is non zero, and the optimal solution to phase I was equal to zero.

(b). The big M method found a finite optimal solution in which some artificial variable is basic, and all artificial variables are equal to zero, and the optimal solution to phase I was equal to zero.

5. (20 points) A company manufactures and sells dog food of two types. Each bag of type 1 dog food contains 2 pounds of lamb and 4 pounds of corn. sells for $10 and costs $5 to produce. Each bag of type 2 dog food contains 1 pound of corn and 1 pound of lamb, sells for $6 and costs $4 to produce. A total of 30 pounds of lamb and 50 pounds of corn are available. All dog food produced can be sold, and the company manager requires that at least 11 bags of dog food 1 are produced. Let \( x_1, x_2 \) be the number of bags of dog food type 1,2 produced. The following LP was formulated and then solved using Lindo to maximize the company’s profit.

Answer each of the following, or explain why you cannot give an answer without rerunning Lindo:

(a). If 46 pounds of corn were available (instead of 50), what would be the new optimal solution to the problem?

(b). What is the most that the company should be willing to pay to for another pound of corn?
(c). The company is considering producing a third type of dog food. Each bag could be sold for $9, would cost $2 to produce, and require 1 pound of lamb and 3 pounds of corn. Should the company produce any type 3 dog food?

(d). The company just learned that 47 pounds of corn are available, and that each bag of dog food 1 can sell for $11 (instead of $10). What would be the new optimal solution to the problem (the $z$)?