AMS 526 Homework 5

Due on Monday 11/21 in class

1. (20 points. Problem 24.4) For an arbitrary matrix $A \in \mathbb{C}^{m \times m}$ and norm $\| \cdot \|$, prove using Theorem 24.9:
   
   (a) $\lim_{n \to \infty} \| A^n \| = 0 \Leftrightarrow \rho(A) < 1$, where $\rho$ is the spectral radius (i.e. $\rho = \max_i \{|\lambda_i|\}$).
   
   (b) $\lim_{t \to \infty} \| e^{tA} \| = 0 \Leftrightarrow \alpha(A) < 0$, where $\alpha$ is the spectral abscissa (i.e. $\rho = \max_i \{\text{Re}(\lambda_i)\}$).

2. (15 points) Problem 25.3 on page 195 of textbook.

3. (20 points) Problem 26.1 on page 200 of textbook.

4. (15 points) Problem 27.1 on page 209 of textbook.

5. (30 points) Write a C code to call LAPACK routines dgesvd and dsyev to perform singular value decomposition and eigenvalue decomposition, and then use them to solve least squares problems $Ax \approx b$. Specifically, your code should use dgesvd to obtain the SVD of $A$ and apply Algorithm 11.3 to solve $Ax \approx b$. In addition, use dsyev to obtain the eigenvalues and eigenvectors of $A^T A$, and then use them to solve the normal equation $A^T A x = A^T b$. You can find the interface definitions for dgesvd and dsyev by searching at [http://netlib.org/cgi-bin/search.pl](http://netlib.org/cgi-bin/search.pl).

Use your C code to solve the least squares problem arising from polynomial fitting of degree $n - 1$,

$$p_{n-1}(t) = x_1 + x_2 t + x_3 t^2 + \cdots + x_n t^{n-1},$$

from $m$ data points $(t_i, y_i)$, $m > n$. Let $t_i = (i - 1)/(m - 1)$, $i = 1, \ldots, m$, so that the data points are equally spaced on the interval [0, 1]. We will generate the corresponding values $y_i$ by first choosing values for the $x_j$, say $x_j = 1$, $j = 1, \ldots, n$, and evaluating the resulting polynomial to obtain $y_i = p_{n-1}(t_i)$, $i = 1, \ldots, m$. The objective is to see whether we can recover the $x_j$ that are used to generate $y_i$. Choose $n = 3, 4, \ldots, 15$ and $m = 2n$, and plot the 2-norm errors using SVD and eigenvalue decomposition.

Submit your modified C code, the plots, and your conclusions of the comparative study.