AMS 526 Homework 6

Due: Friday 12/9 in class

1. (20 points) Question 28.2 on page 218 of textbook.

2. (20 points) Question 30.3 on page 233 of textbook.

3. (20 points) Question 36.1 on page 283 of textbook.

4. (20 points) Let $A \in \mathbb{C}^{m \times m}$ be a Hermitian matrix and $q \in \mathbb{C}^m$ be a vector with $\|q\|_2 = 1$.
   
   (a) Show that there exists a unitary matrix $Q \in \mathbb{C}^{m \times m}$ whose first column is $q$, such that $Q^*AQ$ is a tridiagonal matrix.

   (b) Prove or disprove: There exists a unitary matrix $\tilde{Q} \in \mathbb{C}^{m \times m}$ whose last column is $q$, such that $\tilde{Q}^*AQ$ is a tridiagonal matrix.

5. (20 points plus 10 bonus points)
   (a) (10 points) Suppose that $A$ is an $m \times m$ hermitian matrix. Let $\lambda$ and $\mu$, $\lambda \neq \mu$, be eigenvalues of $A$ with corresponding eigenvectors $x$ and $y$, respectively. Show that $y^*x = 0$ (i.e., show that eigenvectors corresponding to distinct eigenvalues of a hermitian matrix are orthogonal).

   (b) (10 points) Suppose that $A$ is nonhermitian. If $Ax = \lambda x$ and $y^*A = \mu y^*$, with $\lambda \neq \mu$. Show that $y^*x = 0$ (i.e., show that right and left eigenvectors corresponding to distinct eigenvalues are orthogonal).

   (c) (10 bonus points) Suppose that $A$ is non-hermitian. If $Ax = \lambda x$ and $y^*A = \lambda y^*$, where $\lambda$ is a simple eigenvalue. Show that $y^*x \neq 0$ (i.e., show that right and left eigenvectors corresponding to the same simple eigenvalue cannot be orthogonal).