AMS 527, Spring 2016, Homework 5

100 points. Due: Monday 04/04

Electronic submission is required for the programming problems. Please email your programs and the report to the TA. Your email should have the subject line “AMS527 HW#5 Submission”. For the written part, you are encouraged (but not required) to typeset using \LaTeX or LyX (an easy-to-use front-end of \LaTeX). For electronic submission, homework is due at 11:59pm on the due date. For paper submission, homework is due in class on the due date.

1. (10 points) Exercise 9.7 on page 417 of Heath book
   Consider the IVP
   \[ y'' = y \]
   for \( t \geq 0 \), with initial values \( y(0) = 1 \) and \( y'(0) = 2 \).
   (a) Express this second-order ODE as an equivalent system of two first-order ODEs.
   (b) What are the corresponding initial conditions for the system of ODEs in part a?
   (c) Are the solutions of this system stable?
   (d) Perform one step of Euler’s method for this ODE system using a step size of \( h = 0.5 \).
   (e) Is Euler’s method stable for this problem using this step size?
   (f) Is the backward Euler method stable for this problem using this step size?

2. (10 points) Exercise 9.9 on page 417 of Heath book
   For each property listed below, state which of the following three ODE methods has or have the given property.
   (1) \[ y_{k+1} = y_k + \frac{h}{2} \left( f(t_k, y_k) + f(t_{k+1}, y_k + hf(t_k, y_k)) \right) \]
   (2) \[ y_{k+1} = y_k + \frac{h}{2} \left( 3f(t_k, y_k) - f(t_{k-1}, y_{k-1}) \right) \]
   (3) \[ y_{k+1} = y_k + \frac{h}{2} \left( f(t_k, y_k) + f(t_{k+1}, y_{k+1}) \right) \]
   (a) Second-order accurate
   (b) Single-step method
   (c) Implicit method
   (d) Self-starting
   (e) Unconditionally stable
   (f) Runge-Kutta type method
   (g) Good for solving stiff ODE’s

3. (10 points) Exercise 9.10 on page 418 of Heath book
   Use the linear ODE \( y' = \lambda y \) to analyze the accuracy and stability of Heun’s method (see Section 9.3.6). In particular, verify that this method is second-order accurate, and describe or plot its stability region in the complex plane.
   Hint: When finding the stability region, let \( \lambda h = x + iy \), and you may want to introduce a temporary variable \( t = -(x + x^2/2) \).
4. (15 points) Exercise 9.11 on page 418 of Heath book
Applying the midpoint quadrature rule on the interval \([t_k, t_{k+1}]\) leads to the implicit midpoint method
\[
y_{k+1} = y_k + h_k f(t_k + h_k/2, (y_k + y_{k+1})/2)
\]
for solving ODE \(y' = f(t, y)\). Determine the order of accuracy and the stability region of this method.

5. (15 points) Exercise 9.12 on page 418 of Heath book
The centered difference approximation
\[
y' \approx \frac{y_{k+1} - y_{k-1}}{2h}
\]
leads to the explicit two-step, leapfrog method
\[
y_{k+1} = y_{k-1} + 2hf(t_k, y_k)
\]
for solving the ODE \(y' = f(t, y)\). Determine the order of accuracy and the stability region of the method.