AMS 553: Program Set 1

We wish to analyze the following two single-server queueing systems:

(i) An M/M/1 queue, where the interarrival times and service times are independent exponentially distributed random variables

(ii) An M/D/1 queue, where the interarrival times are i.i.d. exponential random variables but the service times are deterministic

In both cases, assume that the mean interarrival time is 5 minutes, and mean service time is 4 minutes (in case (ii), this implies that the service time is exactly 4 minutes).

Write a computer simulation program to simulate the above systems (or two separate programs, one for each of the two systems). Your program should be able to calculate the following performance measures:

- average time that a customer waits in queue
- average number of customers in the queue
- fraction of customers that spent more than 4.5 minutes in the system
- fraction of time that the system is busy

In your program, use the LCG

\[ Z_{i+1} = 16807Z_i \mod 2147483647, \quad U_{i+1} = Z_{i+1}/2147483647, \quad i = 0, 1, 2, \ldots \]

...to generate random numbers. Use two separate streams of random numbers to generate interarrival times and service times. In particular, use the initial seed \( Z_0 = 1155192168 \) for interarrival times and \( Z_0 = 1806794933 \) for service times. To generate i.i.d. exponential random variates \( X_i \sim \text{exp}(\lambda) \) (\( \lambda \) is the parameter of the distribution), use the following generator

\[ X = \frac{-1}{\lambda} \ln(U). \]

Your program should be able to handle two stopping conditions: (1) a given number of customers completing service; (2) stop at some given fixed simulation time. For each of the two queueing systems, run your simulation program under the following stopping conditions (one replication for each stopping condition):

1. 10000 customers completing service
2. 1000 hours of simulation time

assuming that the system starts empty, and calculate the four performance measures for all cases.

Hint: (1) You may want to consult Chapter 1.4 of your textbook. (2) To ensure that your program is implemented correctly, run short simulations with the initial seeds given above, do a hand calculation and compare the result with your computer program output.