Due at the beginning of class on Thursday, October 17, 2013. Reminder: Show your reasoning!

Read: Ross, Chapter 4, Sections 4.5–4.10 (you can skip the Hypergeometric distribution (4.8.3) and the Zeta distribution (4.8.4)), Sections 5.1–5.3.

SUBMIT SOLUTIONS TO 4 OF THE 5 PROBLEMS BELOW. You are responsible to be able to solve all 5 of them.

**(1).** *(25 points)* Fifteen percent of houses in your area have finished basements. Your real estate agent starts showing you homes at random, one after the other. Let $X$ be the number of homes with finished basements that you see, before the first house that has no finished basement.

(a). *(5 points)* What is the distribution of $X$? (If it has a special name, give it (along with any parameters). Also, give its pmf.)

(b). *(5 points)* What is the distribution of $Y = X + 1$? (If it has a special name, give it (along with any parameters). Also, give its pmf.)

(c). *(5 points)* What is the mean and the variance of $X$?

(d). *(10 points)* Assume now that you see 400 homes on Monday. Let $Z$ be the number of them that had finished basements.

(i). *(5 points)* What is the (exact) probability that you see fewer than 70 homes with finished basements? (you need not evaluate any arithmetic expression)

(ii). *(5 points)* What does the Poisson approximation give you as an approximate probability that you see fewer than 70 homes with finished basements? (you need not evaluate any arithmetic expression)

**(2).** *(25 points)* The amount of time (in hours) that Joe spends checking email each day is a random variable $X$. A day is a “heavy email day” if Joe spends at least 45 minutes on checking email that day. Assume that the density function of $X$ is given by

$$f(x) = \begin{cases} \frac{x^2}{2} & \text{if } 0 \leq x < 1 \\ C & \text{if } 2 < x < 3; \\ 0 & \text{otherwise.} \end{cases}$$

(a). *(5 points)* What is the probability that Joe spends more than 15 minutes checking email on Thursday?

(b). *(5 points)* On average, how many minutes does Joe spend each day checking email?

(c). *(5 points)* Compute $P(0.75 < X \leq 2.2 \mid X > 0.5)$

(d). *(5 points)* Compute $P(X = 2), P(X = 0), P(X = E(X)), P(X > E(X))$, and $F(E(X))$.

(e). *(5 points)* What is the probability that there are an odd number of heavy email days in October?
(3). (25 points) The density function of $X$ is given by
\[ f(x) = \begin{cases} 
  ax + bx^3 & \text{if } 0 \leq x \leq 1 \\
  0 & \text{otherwise}
\end{cases} \]
Suppose also that you are told that $E(X) = 3/5$. (a). (5 points) Find $a$ and $b$; (b). (10 points) Determine the cdf, $F(x)$, explicitly; (c). (10 points) Determine the pdf, $f_Y(y)$, of $Y = X^2 + 1$.

(4). (25 points) [Related to Examples 2a and 2b (Ross, Chap 5).] Let $X$ be the length of time it takes Joe to grade the midterm, measured in tens of hours. The cdf of $X$ is given by
\[ F(x) = \begin{cases} 
  1 - \frac{16}{x^4} & \text{if } x \geq 2 \\
  0 & \text{if } x < 2
\end{cases} \]
(a). Find the probability density function for $X$; be explicit about all cases.
(b). How long does Joe expect to take grading the midterm?
(c). It is now 7:00am and Joe has been grading nonstop for 2 full days (night and day). What is the probability that Joe is done grading in time to hand the midterms back at 12:50pm today?

(5). (25 points) [Related to Examples 5a, 5b, 5d (Ross).] Joe keeps changing jobs. Assume that the time between when Joe starts one job and when he starts the next job is an exponential random variable. There is a 10% chance that Joe will stay at a job more than 3 years.
(a). Joe has been working at Stony Brook for 5 years; what is the chance that Joe will still be working at Stony Brook 7 years from now?
(b). It is 7 years later, and Joe is still working at Stony Brook all of these 12 years. What is the probability that Joe will leave Stony Brook for another job within the next 12 months?
(c). It is October of 2013, and Joe has been at Stony Brook for 13 years. What is the expected date of his starting a new job (the first one after Stony Brook)?
(d). Joe is one of 5 siblings, all of them working at Stony Brook at the moment (October 17, 2013), and all of them changing jobs, independently, according to the same probability distribution assumptions given above (they each spend an exponential amount of time between jobs, and have a 10% chance of being at a job more than 3 years). What is the probability that at least 3 of the 5 siblings will have left Stony Brook for new jobs by December 17, 2014?