Operations Research II: Stochastic Models  
Homework Set # 1

Due at the beginning of class on Wednesday, January 31, 2007. Reminder: Show your reasoning!

Recommended Reading: Ross, Chapter 1; Chapter 2, Sections 2.1–2.5, 2.8;

Note: All problem numbers refer to the text, Ross, Ninth Edition. (If you use an earlier edition of the text, you will have to determine the correct problems to do.)

(1). (12 points) # 21, Chapter 1: Suppose that 5 percent of men and 0.25 percent of women are color-blind. A color-blind person is chosen at random. What is the probability of this person being male? Assume that there are an equal number of males and females.

(2). (12 points) # 26, Chapter 1: A deck of 52 playing cards, containing all 4 aces, is randomly divided into 4 piles of 13 cards each. Define events $E_1$, $E_2$, $E_3$, and $E_4$ as follows: $E_i = \{ \text{the } i\text{th pile has exactly 1 ace} \}$. Determine the probability $P(E_1 \cap E_2 \cap E_3 \cap E_4)$ that each pile has an ace. (Hint: see exercise 23 of Chapter 1.)

(3). (12 points) # 39, Chapter 1: Stores $A$, $B$, and $C$ have 50, 75, and 100 employees, and respectively, 50, 60, and 70 percent of these are women. Resignations are equally likely among all employees, regardless of sex. One employee resigns and this is a woman. What is the probability that she works in store $C$?

(4). (12 points) # 7, Chapter 2: Suppose a coin having probability 0.7 of coming up heads is tossed three times. Let $X$ denote the number of heads that appear in the three tosses. Determine the probability mass function of $X$. Be very explicit! (give $p_X(x)$ for all values of $x$.)

(5). (12 points) # 9, Chapter 2: If the distribution function of $F$ is given by

$$F(b) = \begin{cases} 
0, & b < 0 \\
1/2, & 0 \leq b < 1 \\
3/5, & 1 \leq b < 2 \\
4/5, & 2 \leq b < 3 \\
9/10, & 3 \leq b < 3.5 \\
1 & b \geq 3.5 
\end{cases}$$

calculate the probability mass function of $X$.

(6). (12 points) # 13, Chapter 2: An individual claims to have extrasensory perception (ESP). As a test, a fair coin is flipped ten times, and he is asked to predict in advance the outcome. Our individual gets seven out of ten correct. What is the probability he would have done at least this well if he had no ESP? (Explain why the relevant probability is $P(X \geq 7)$ and not $P(X = 7)$.)

(7). (14 points) # 34, Chapter 2: Let the probability density of $X$ be given by

$$f(x) = \begin{cases} 
c(4x - 2x^2) & 0 < x < 2 \\
0 & \text{otherwise} 
\end{cases}$$

(a). What is the value of $c$? (b). Compute $P(\frac{1}{2} < X < \frac{3}{2})$. Also compute the expected value $E(X)$.

(8). (14 points) # 40, Chapter 2: Suppose that two teams are playing a series of games, each of which is independently won by team $A$ with probability $p$ and by team $B$ with probability $1 - p$. The winner of the series is the first team to win four games. Find the expected number of games that are played, and evaluate this quantity when $p = 1/2$. 