Due at the beginning of class on Wednesday, March 21, 2007. Reminder: You must show your reasoning for full credit!

Recommended Reading: Ross, Chapter 5, Section 5.4; Chapter 6, Sections 6.1-6.3

(1). [15 points] The number of hours between successive train arrivals at the station is uniformly distributed on (0,1). Passengers arrive according to a Poisson process with rate 7 per hour. Suppose a train has just left the station. Let \( X \) denote the number of people who get on the next train. Find (a) \( E(X) \), and (b) \( \text{var}(X) \).

(2). [15 points] In a certain system, a customer must first be served by server 1 and then by server 2. The service times at server \( i \) are exponential with rate \( \pi_i \), \( i = 1, 2 \). An arrival finding server 1 busy waits in line for that server. Upon completion of service at server 1, a customer either enters service with server 2 if that server is free or else remains with server 1 (blocking any other customer from entering service) until server 2 is free. Customers depart the system after being served by server 2. Suppose that when you arrive there is one customer in the system and that customer is being served by server 1. What is the expected total time you spend in the system?

(3). [20 points] Assume that vehicles arrive at a toll plaza according to a Poisson process. The average inter-arrival time between vehicles is 10 minutes. One third of the vehicles are cars, and two thirds are trucks. (And the event that a vehicle is a car is independent of the arrival process.) The lunch hour is from 12:00 to 1:00. Find
   (a). \( P\{ \text{at least 8 cars arrived in [12:03, 12:07]} \mid 10 \text{ cars arrived during the lunch hour} \} \)
   (b). \( P\{ \text{at least 2 trucks arrived in [12:10, 12:30]} \mid 20 \text{ cars arrived during lunch} \} \)
   (c). \( \text{var}\{ \text{number of trucks that arrive in [11:00, 1:20]} \} \)
   (d). Assume now that vehicles arrive more frequently during the lunch hour: the average inter-arrival time is 5 minutes during the lunch hour (and 10 minutes otherwise). Answer part (c) for this new assumption.

(4). [10 points] The water level of a certain reservoir is depleted at a constant rate of 1000 units daily. The reservoir is refilled by randomly occurring rainfalls. Rainfalls occur according to a Poisson process with rate 0.2 per day. The amount of water added to the reservoir by rainfall is 5000 units with probability 0.8 or 8000 units with probability 0.2. The present water level is just slightly below 5000 units.
   (a). What is the probability the reservoir will be empty after 5 days?
   (b). What is the probability the reservoir will be empty some time within the next 10 days?

(5). [20 points] # 1, Chapter 6. (same in Ross 8th or 9th edition)

(6). [20 points] Consider two machines that are maintained by a single repairman. Machine \( i \) functions for an exponential time with rate \( \mu_i \) before breaking down, \( i = 1, 2 \). The repair times (for either machine) are exponential with rate \( \mu \). Can we analyze this as a birth and death process? If so, what are the parameters? If not, how can we analyze it as a CTMC? (give the states (precisely), the holding time parameters, and the transition (jump) matrix)