AMS310, Lecture 4, Summer 2003

Continuous random variable

- cumulative density function $F(x) = P(X < x)$
- probability density function (pdf) $\int_{-\infty}^{x} f(x)dx = F(x)$, $F'(x) = f(x)$
- geometric representation of $P(a < X < b) = \int_{a}^{b} f(x)dx = F(b) - F(a)$.
- the mean (expectation) of a random variable (a probability distribution)
  \[ \mu = \int_{-\infty}^{\infty} x \cdot f(x)dx \]
- the variance of a random variable
  \[ \sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 \cdot f(x) \]

Probability models and distributions:

- Uniform(a, b)
  - $f(x) = \frac{1}{b-a}$, $a \leq x \leq b$
  - $\mu = \frac{a+b}{2}$, $\sigma^2 = \frac{1}{12}(b-a)^2$
- Normal distribution: the most important distribution, also called Gaussian distribution.
  - $f(x) = \frac{1}{\sqrt{2\pi}\sigma^2} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$
  - two parameters: mean $\mu$, variance $\sigma$. Normal distribution is then denoted by $N(\mu, \sigma^2)$.
  - Standard Normal, when $\mu = 0$, $\sigma = 1$, $N(0, 1)$
  - Table 3 on page 586 gives the CDF of $N(0, 1)$
  - If $X$ is $N(\mu, \sigma^2)$, $Z = \frac{X - \mu}{\sigma}$ is $N(0, 1)$. Therefore, we can use Table 3 to find probabilities associated with any normal distribution.
- Exponential distribution: describes waiting time between events
- Weibull distribution: Models lifetime data
- Gamma distribution: a large family of distribution with includes exponential and $\chi^2$-distribution as its special cases.
  - $\mu = \lambda$, $\sigma^2 = \lambda$
- Beta distribution, models the random variables which take values between 0 and 1.

Homework: 5.2, 5.8, 5.14, 5.24, 5.34, 5.46, 5.38, 5.40, 5.52, 5.58