AMS310, Lecture 6, Summer 2003

Point Estimates

- Estimator: A statistic intended for estimating a parameter e.g. \( \bar{X}, S^2 \)
- estimate: An observed value of the estimator
- standard error(s.e.): standard deviation of an estimator, \( s.e.(\bar{X}) = \sigma/\sqrt{n} \)
- An estimator \( (\hat{\theta}) \) is an unbiased estimator of \( \theta \) if \( E(\hat{\theta}) = \theta \). \( E(\hat{\theta}) - \theta \) is called bias.

Confidence Interval of \( \mu \):

- maximum error: how close is an estimate from the true parameter.
  - normal with \( \sigma \) known: \( E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \)
  - large sample size with \( \sigma \) known: \( E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \)
  - normal with \( \sigma \) unknown: \( E = t_{\alpha/2} \cdot \frac{s}{\sqrt{n}} \)

- Confidence Interval with \((1-\alpha)\) confidence: \( \bar{X} \pm E \) or \( (\bar{X} - E, \bar{X} + E) \)

- Interpretation of C.I.: if the sampling procedure is repeated many many times, \((1-\alpha)\) of the confidence intervals based on the sample data will cover the true mean \( \mu \).

- Values of \( z_{\alpha/2} \):

<table>
<thead>
<tr>
<th>( 1 - \alpha )</th>
<th>0.80</th>
<th>0.90</th>
<th>0.95</th>
<th>0.99</th>
</tr>
</thead>
<tbody>
<tr>
<td>( z_{\alpha/2} )</td>
<td>1.28</td>
<td>1.645</td>
<td>1.96</td>
<td>2.58</td>
</tr>
</tbody>
</table>

- sample size needed to attain maximum error: \( n = \left( \frac{z_{1 - \alpha} \sigma}{E} \right)^2 \)

Hypothesis testing:

- Null Hypothesis: the naive assertion;
- Alternative Hypothesis: the assertion you want to approve.
- Analogy to the Justice system: keep the null hypothesis unless there is an strong evidence against it.
- Evidence: Data

- Measure of the strength of the evidence: P-value, the probability of observing more extreme or as extreme as the observed under the null hypothesis.

- Type I error: reject \( H_0 \) when \( H_0 \) is true.
- Type II error: retain \( H_0 \) when \( H_0 \) is false.

**t-distribution**

Homework: 7.6, 7.8, 7.12, 7.22