Applied Calculus III

Practice Midterm II

1) Sketch the region over which the integral

\[ \int_{1}^{4} \int_{-\sqrt{y}}^{\sqrt{y}} f(x, y) \, dx \, dy \]

is performed.

2) Set up the integral of the function \( f(x, y) \) over the indicated region bounded by the functions \( g(x) \) and \( h(x) \). Do not attempt to evaluate the integral.

![Graph of the region](image)

3) Under the change of variables \( s = 3x + y, t = x - 4y \), the region \( \mathcal{R} \) in the \( xy \) plane is mapped to the region \( \mathcal{W} \) in the \( st \) plane. Rewrite

\[ \int_{\mathcal{R}} \cos \left( \frac{x - 4y}{3x + y} \right) \, dx \, dy \]

as an integral in \( s, t \). Do not attempt to evaluate the integral you obtain.

4) Compute the integral

\[ \int_{0}^{1} \int_{3}^{4} \sin(2 - y) \cos(3x - 7) \, dx \, dy \]

5) By a change to either cylindrical or spherical coordinates, as appropriate, evaluate

\[ \int_{-\sqrt{3}}^{\sqrt{3}} \int_{-\sqrt{3-x^2}}^{\sqrt{3-x^2}} \int_{1}^{4-x^2-y^2} \frac{1}{z^2} \, dz \, dy \, dx \]

6) Find the parametrization for the line through the point \((1,3,2)\) perpendicular to the \( yz \)-plane.
7) Find the parametrization for the circle of radius 3 parallel to the $xz$-plane, centered at the point $(0,5,0)$ and traversed counterclockwise when viewed from $(0,10,0)$.
8) Consider the curve $x^2z = 1$ in the $xz$-plane. Obtain a parametrization of the surface obtained by rotating this curve around the $z$-axis, for $z > 0$.
9) Solve for the flow line of the vector field

$$ \vec{F} = e^y \hat{i} + \frac{1}{2} \hat{j} $$

passing through the point $(0,1)$ at $t = 0$.
10) a) Sketch the vector field

$$ \vec{F} = \left( \frac{x^2}{x^2 + y^2} \right) \hat{i} + \left( \frac{y^2}{x^2 + y^2} \right) \hat{j} $$

for a selection of points on the $x$ and $y$ axes and along the lines $y = \pm x$. Ignore the origin $(0,0)$. b) Write the differential equation for the flow line of the vector field passing through the point $(4,2)$ at $t = 0$. Do not try to solve the differential equations.
Practice Midterm II - Answers

1)

2)

\[ \int_{-2}^{3} \int_{g(x)}^{h(x)} f(x, y) dy dx \]

3)

\[ \frac{1}{13} \int_{\mathcal{V}} \cos \left( \frac{t}{s} \right) ds dt \]

4)

\[ \left( \frac{\sin(5) - \sin(2)}{3} \right) (\cos(1) - \cos(2)) \]

5)

\[ \int_{1}^{4} \int_{0}^{2\pi} \int_{0}^{\sqrt{4-z}} \frac{1}{z^2} r dr d\theta dz = \pi [3 - \ln 4] \]

6)

\[ \vec{r}(t) = (1 + t)\hat{i} + 3\hat{j} + 2\hat{k} \]

7)

\[ \vec{r}(\theta) = 3 \cos \theta \hat{i} + 5\hat{j} + 3 \sin \theta \hat{k}, \quad 0 \leq \theta < 2\pi \]
8) \[ \vec{r}(t) = \frac{1}{\sqrt{z}} \cos \theta \hat{i} + \frac{1}{\sqrt{z}} \sin \theta \hat{j} + z \hat{k}, \quad z > 0, \quad 0 \leq \theta < 2\pi \]

9) \[
\begin{align*}
\frac{dx}{dt} &= e^{\nu(t)} \quad y(t) = \frac{t}{2} + 1 \\
\frac{dy}{dt} &= 1/2 \quad x(t) = 2e^{t/2+1} - 2e = 2e^{t/2} - 1
\end{align*}
\]

10a) 

b) \[
\begin{align*}
\frac{dx}{dt} &= \frac{x^2}{x^2 + y^2} \\
\frac{dy}{dt} &= \frac{y^2}{x^2 + y^2} \\
x(t = 0) &= 4, \quad y(t = 0) = 2
\end{align*}
\]