1. (10 points) (Bouncing Ball Problem)
Consider the equation
\[ y'' + b|y'|y' + g = q(y) \] on \((0, 2)\).

Suppose that \(y(0) = 1\), \(y'(0) = 0\), \(g = 10\). The function \(q(y)\) is zero for \(y\) nonnegative and \(-10^6y\) for \(y\) negative.
If we let \(E(t) = gy + 0.5(y')^2 + Q(y)\), where \(Q(y)\) is zero for \(y\) nonnegative and \(5 \times 10^5 \times y^2\) for \(y\) negative. Then
\[ E' = -b|y'|^3. \]

Using \(Y = (y, y', E)^T\), the above relations can be expressed as an initial value problem. Use Heun’s method to solve this problem.
Implement for time step size \(h = 0.0001\) and \(b = 0.01, 0.001, 0.0001\).
Produce three plots \((y, t)\), \((y', t)\), and \((E, t)\) on \((0, 2)\) for the above \(h\) and \(b\).

2. (10 points) Since
\[ \int_0^1 \frac{4}{1 + x^2} \, dx = \pi, \]
one can compute an approximate value for \(\pi\) using numerical integration of the given function.
Use the midpoint, trapezoid, and Simpson composite quadrature rules to compute the approximate value for \(\pi\) for numbers of subintervals 2, 4, 8, 16, 32, 64, 128, 256. Compute the errors and order of accuracy.

3. (5 points) The temperature dependence of reaction rate coefficient of a chemical reaction is often modeled by the Arrhenius equation
\[ k = A \exp(-E_a/RT), \]
where \(k\) is the reaction rate, \(A\) is the preexponential factor, \(E_a\) is the activation energy, \(R\) is the universal gas constant, and \(T\) is the absolute temperature. Experimental data for a particular reaction yield the following results: Use a least-squares fit of this data to obtain values for \(A\) and \(E_a\) for the reaction. Take \(R = 8314\). Plot the least square fit with solid line and data with “o”.