AMS312 Spring 2005
Quiz 5 Solution

1. Please see the textbook 7.5.13 (Page438).

7.5.13 Let \( \mu \) be the true average GMAT increase earned by students taking a review course. The hypotheses to be tested are \( H_0: \mu = 40 \) versus \( H_1: \mu < 40 \). Here, \( \sum_{i=1}^{15} \bar{y}_i = 556 \) and

\[
\sum_{i=1}^{15} y_i^2 = 20,966, \quad \bar{y} = \frac{556}{15} = 37.1, \quad s = \sqrt{\frac{15(20,966) - (556)^2}{15(14)}} = 5.0, \quad \text{and} \quad t = \frac{37.1 - 40}{5.0/\sqrt{15}} = -2.25.
\]
Since \(-t_{0.05,14} = -1.7613\), \(H_0\) should be rejected at the \( \alpha = 0.05 \) level of significance, suggesting that the MBAs 'R Us advertisement may be fraudulent.

2. Please see the textbook 7.5.1 (Page433).

7.5.1 Given that \( n = 7 \), \( t_{0.25,6} = t_{0.25,6} = 2.4469 \). Here

\[
\bar{y} = \frac{1}{n} \sum_{i=1}^{n} y_i = \frac{1}{7}(12808) = 1829.71,
\]
so the 95% confidence interval for \( \mu \) reduces to

\[
\left( 1829.71 - \frac{2.4469 \cdot 719.43}{\sqrt{7}}, 1829.71 + \frac{2.4469 \cdot 719.43}{\sqrt{7}} \right) = (\$1164, \$2495).
\]

3. Please see the textbook 7.3.14 (Page420).

7.3.14 \( P\left( \chi^2_{2, a - 1} \leq \frac{(n - 1)S^2}{\sigma^2} \leq \chi^2_{1 - a / 2, a - 1} \right) = 1 - \alpha = P\left( \frac{(n - 1)S^2}{\chi^2_{1 - a / 2, a - 1}} \leq \frac{(n - 1)S^2}{\chi^2_{2, a - 1}} \right) \), so

\[
\left( \frac{(n - 1)S^2}{\chi^2_{1 - a / 2, a - 1}}, \frac{(n - 1)S^2}{\chi^2_{2, a - 1}} \right)
\]
is a 100(1 - \( \alpha \))% confidence interval for \( \sigma^2 \). Taking the square root of both sides gives a 100(1 - \( \alpha \))% confidence interval for \( \sigma \).

4. Please see the textbook 7.3.16 (Page420).

7.3.16 a) \( \sum_{i=1}^{18} y_i = 1447 \), so

\[
\bar{y} = \frac{1447}{18} = 80.4 \quad \text{and} \quad s = \frac{1}{17} \sum_{i=1}^{18} (y_i - 80.4)^2 = 5.1.
\]
Since \( \chi^2_{0.05,17} = 7.564 \) and \( \chi^2_{0.75,17} = 30.191 \), a 95% confidence interval for \( \sigma^2 \) is

\[
\left( \frac{17(5.1)^2}{30.191}, \frac{17(5.1)^2}{7.564} \right),
\]
or \((3.8, 7.6)\).

b) Given that \( \chi^2_{0.05,17} = 8.762 \) and \( \chi^2_{0.75,17} = 27.587 \), the two one-sided confidence intervals for \( \sigma \) are

\[
(-\infty, \sqrt{\frac{17(5.1)^2}{8.672}}) = (-\infty, 7.1) \quad \text{and} \quad \left( \sqrt{\frac{17(5.1)^2}{27.587}}, \infty \right) = (4.0, \infty).
\]

Note: (1) For problem 3, if a student did not show

\[
P\left( \frac{(n - 1)S^2}{\sigma^2} \leq \chi^2_{1 - a / 2, a - 1} \right) = 1 - \alpha, \quad 5 \text{ points will be subtracted.}
\]

(2) For 4(b) part, both \((-\infty, 7.1)\) and \((0, 7.1)\) are right.