1. A survey conducted 5 years ago concluded that the average U.S. adult heights was 68 inches. However, it is suspected that the average U.S. adult heights shorter than 68 inches. A random sample of 25 U.S. adults is drawn. The sample mean (the average height of these 25 adults) is found to be 67.4 inches, and the sample standard deviation is 6 inches. At the significance level $\alpha = 0.05$, can you conclude that average U.S. adults height nowadays is shorter than 68 inches? What is the $p$-value?

SOLUTION

(a) $H_0 : \mu = 68 \text{ inches} \quad H_a : \mu < 68 \text{ inches}.$
(b) $\bar{x} = 67.4 \text{ inches}, \ n = 400$. Under $H_0 : \mu = 68 \text{ inches}(\mu_0)$, the test statistic is

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{67.4 - 68}{6/\sqrt{25}} = \frac{-0.6}{1.2} = -0.5$$

(c) The traditional method: Because $|t| = 0.5 < 1.711$ ($= t_{0.05,24}$), we cannot reject $H_0$.
(d) The $p$-value $= P(\bar{X} \leq 67.4) = P(t \leq -0.5) > 0.3108$ (shaded area).
2. Jerry is planning to purchase a sporting goods store in a small city. He calculated that in order to cover basic expenses, average daily sales must be at least $525. He checked the daily sales of 26 randomly selected business days. And he found that the average daily sales for those 26 days was $565 with a standard deviation of $150. At the significance level \( \alpha = 0.05 \), can Jerry conclude that the average daily sales is higher than $525? What is the \( p \)-value?

**SOLUTION**

(a) \( H_0 : \mu = 525 \quad H_a : \mu > 525 \).

(b) \( \bar{x} = 565, \; s = 150, \; n = 26 \). Under \( H_0 : \mu = 525 \), the test statistic is

\[
t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{565 - 525}{150/\sqrt{26}} = \frac{40}{150/\sqrt{26}} = 1.36
\]

(c) The traditional method: Because \( t = 1.36 < 1.708 (= t_{0.05,25}) \), we cannot reject \( H_0 \) at \( \alpha = 0.05 \). In other words, it is likely that the average daily sales may not be higher than $525. Therefore it is risky to purchase this store.

(d) \[ p \text{-value} = P(\bar{X} \geq 565) = P(t \geq 1.36) = 0.0929. \] (Shaded area)

Since \( p \)-value > 0.05, we fail to reject \( H_0 \).
3. The nighttime cold medicine Dozenol bears a label indicating the presence of 600 mg. of acetaminophen in each fluid ounce of the drug. The Food and Drug Administration (FDA.) randomly selected 15 1-oz. samples and found that the mean acetaminophen content is 593 mg., whereas the standard deviation is 21 mg. Using $\alpha = 0.05$, test the claim of the Medassist Pharmaceutical Company that the population mean is equal to 600 mg. What is the $p$-value? Construct a 95% confidence interval for $\mu$.

**SOLUTION**

(a) $H_0 : \mu = 600 \text{ mg}$  \hspace{1em} $H_a : \mu \neq 600 \text{ mg}$.

(b) $\bar{x} = 593 \text{ mg}$, $s = 21 \text{ mg}$, $n = 15$. Under $H_0 : \mu = 600 \text{ mg}$, the test statistic is

$$
t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{593 - 600}{21/\sqrt{15}} = -1.291 \hspace{1em} t_{0.025,14} = 2.145
$$

(c) The traditional method: Because $t = -1.291 > -2.145 (= -t_{0.025,14})$, we cannot reject the null hypothesis at $\alpha = 0.05$.

(d) $p$-value $= 2 \times P(\bar{X} \leq 593) = 2 \times \Phi(-1.291) = 2 \times 0.109 = 0.218$.

(e) A 95% confidence interval for $\mu$ is $(\bar{x} - t_{\alpha/2,14}, \bar{x} + t_{\alpha/2,14} \cdot \frac{s}{\sqrt{n}}) = (593 - 2.145 \cdot \frac{21}{\sqrt{15}}, 593 + 2.145 \cdot \frac{21}{\sqrt{15}}) = (591.37, 604.63)$. 

![Rejection Region](image-url)