AMS 572.01 Midterm
Fall, 2002

Instructions.
This is a closed book and closed notes exam. Please provide detailed solutions. Anyone who cheats on the exam shall receive a score of zero.

1. A sample of 100 married couples was taken from a certain population. Husbands and wives were interviewed separately to determine whether their main source of news was from the newspaper, or radio/television. The results may be found in the following table.

<table>
<thead>
<tr>
<th></th>
<th>Husband</th>
<th>Wife</th>
</tr>
</thead>
<tbody>
<tr>
<td>papers</td>
<td>radio/TV</td>
<td></td>
</tr>
<tr>
<td>papers</td>
<td>15</td>
<td>16 (b)</td>
</tr>
<tr>
<td>radio/TV</td>
<td>24 (c)</td>
<td>45</td>
</tr>
</tbody>
</table>

Please test whether the distribution of information source is the same for the wives and the husbands. Use $\alpha = 0.05$.
(Sol) This is inference on 2 population proportions, paired samples, and thus McNemar’s test is suitable. The hypotheses are

$$H_0 : p = \frac{1}{2} \text{ v.s. } H_a : p \neq \frac{1}{2}$$

Since both $b$ and $c$ are larger than 5, the large sample test with continuity correction can be applied to simplify the calculation as follows:

$$p_L = P(B \leq 16 | B + C = 40) \approx P\left(Y \leq 16 + \frac{1}{2}\right)$$

where

$$B \sim Bin\left(m, p = p_0 = \frac{1}{2}\right), \quad Y \sim N\left(\mu = mp_0, mp_0(1 - mp_0)\right)$$

Therefore

$$p_L = P\left(\frac{Y - mp_0}{\sqrt{mp_0(1 - mp_0)}} \leq \frac{b + \frac{1}{2} - mp_0}{\sqrt{mp_0(1 - mp_0)}}\right)$$

$$z_L = \frac{b + \frac{1}{2} - mp_0}{\sqrt{mp_0(1 - mp_0)}} = \frac{b - mp_0 + \frac{1}{2}}{\sqrt{mp_0(1 - mp_0)}} = \frac{b - c + 1}{\sqrt{b + c}} = -1.1068$$

and thus the lower one-sided $p$-value is $p_L = \Phi(z_L) = 0.134191$. Also we can find the upper one-sided $p$-value $p_U$ by

$$p_U = P(B \geq 16 | B + C = 40) \approx P\left(Y \geq 16 - \frac{1}{2}\right)$$

$$z_U = \frac{b - \frac{1}{2} - mp_0}{\sqrt{mp_0(1 - mp_0)}} = \frac{b - mp_0 - \frac{1}{2}}{\sqrt{mp_0(1 - mp_0)}} = \frac{b - c - 1}{\sqrt{b + c}} = -1.42302.$$  

The corresponding one-sided upper $p$-value by $p_U = 1 - \Phi(z_U) = 0.922636$. The two-sided $p$-value is $2 \min(p_L, p_U) = 2 \times 0.134191 = 0.268$ which is greater than 0.05. Therefore we cannot reject the null hypothesis.

2. A Gallup survey portrays U.S. entrepreneurs as “... the mavericks, dreamers, and loners whose rough edges and uncompromising need to do it their own way set them in sharp contrast to senior executives in major American corporations” (Wall Street Journal, May 1985). One of the many questions put to a sample of 100 entrepreneurs about their work habits, social activities, etc., concerned the origin of the car they personally drive most frequently. The responses are given in the following table.

<table>
<thead>
<tr>
<th></th>
<th>U.S.</th>
<th>Europe</th>
<th>Japan</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>45</td>
<td>46</td>
<td>9</td>
</tr>
</tbody>
</table>
Do these data provide evidence of a difference in the preference of entrepreneurs for domestic cars versus foreign cars? Testing using $\alpha = 0.05$.

This is inference on one population proportion, large sample. Let $p$ be the population proportion of domestic cars, we have

$$\hat{p} = \frac{45}{100}$$

the test hypotheses are

$$H_0 : p = \frac{1}{2} \text{ v.s. } H_a : p \neq \frac{1}{2}$$

The test statistic is

$$z_0 = \frac{\hat{p} - \frac{1}{2}}{\sqrt{\frac{1}{2}(1 - \frac{1}{2})/100}} = -1.000 > -1.96.$$ 

Therefore, we cannot reject $H_0$.

3. Bear gallbladder has been used in Chinese medicine for more than 1,300 years. Today it is used to treat a wide variety of ailments, including gallstones and other liver diseases. Due to the difficulty of obtaining bear gallbladder, Chinese medical researchers are searching for a more readily available source of animal bile. A study in the Journal of Ethnopharmacology (June 1995) examined pig gallbladder as an effective substitute for bear gallbladder. Twenty mice were divided randomly into two groups: 10 were given a dosage of bear bile and 10 were given a dosage of pig bile. All the mice then received an injection of croton oil in the left ear lobe to induce inflammation. Four hours later, both the left and right ear lobes were weighted, with the difference (in milligrams) representing the degree of swelling. Summary statistics on the degree of swelling are provided in the following table. Please test whether the pig bile is as effective as the bear bile at $\alpha = 0.05$.

<table>
<thead>
<tr>
<th>Bear Bile</th>
<th>Pig Bile</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n_1 = 10$</td>
<td>$n_2 = 10$</td>
</tr>
<tr>
<td>$\bar{x}_1 = 9.19$</td>
<td>$\bar{x}_2 = 9.71$</td>
</tr>
<tr>
<td>$s_1 = 4.17$</td>
<td>$s_2 = 3.33$</td>
</tr>
</tbody>
</table>

This is inference on two population means, independent but small samples. First we need to check whether the population variances are equal or not. The test statistic is $F_0 = s_1^2/s_2^2 = 1.56814$. Since $n_1 = n_2 = 10$ and thus the d.f.s of the $F$ distribution are $(9, 9)$ and the upper 2.5th percentile is $F_{9,9,0.025} = 4.02399$. Since $F_0 < F_{9,9,0.025}$, we may assume that $\sigma_1^2 = \sigma_2^2$ and the pooled-variance $t$-test can be used for testing the equality of the population means. The hypothesis for testing whether pig biles are as efficient as bear biles is $H_0 : \mu_1 = \mu_2 \text{ v.s. } H_a : \mu_1 < \mu_2$. The test statistic is

$$t_0 = \frac{\bar{x} - \bar{y} - 0}{S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

where $S_p^2 = \frac{9s_1^2 + 9s_2^2}{18} = \frac{s_1^2 + s_2^2}{2}$

$$= -0.308141.$$ 

$t_{n_1 + n_2 - 2,0.05} = t_{18,0.05} = 1.734$ and $t_0 > -1.734$. We cannot reject $H_0$. Therefore we conclude that the pig bile seems to be as effective as the bear bile.

4. (extra credit). Please derive Fisher’s exact test. (Solution was given in class.)