Solution for HW of Chapter 9

9.6  (a) For sensitivity, \( \hat{p} = 0.8 \) since 80 out of 100 high risk patients were correctly identified. Then a 90% CI for the sensitivity of the test is given by

\[
\hat{p} \pm 1.645 \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.8 \pm 1.645 \sqrt{\frac{(0.8)(0.2)}{100}} = [0.734, 0.866].
\]

(b) For specificity, \( \hat{p} = 0.92 \) since 184 out of 200 non high risk patients were correctly identified. Then a 90% CI for the specificity of the test is given by

\[
\hat{p} \pm 1.645 \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.92 \pm 1.645 \sqrt{\frac{(0.92)(0.08)}{200}} = [0.888, 0.952].
\]

9.12 The hypotheses are

\[ H_0 : p_1 = p_2 \text{ vs. } H_1 : p_1 \neq p_2. \]

For the vitamin C group, the proportion catching cold is \( \hat{p}_1 = 17/139 = 0.122 \). For the placebo group, the proportion catching cold is \( \hat{p}_2 = 31/140 = 0.221 \). Then the test statistic is

\[
z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}} = \frac{0.122 - 0.221}{\sqrt{\frac{(0.122)(0.878)}{139} + \frac{(0.221)(0.779)}{140}}} = -2.212.
\]

The \( P \)-value is

\[ P = 2(1 - \Phi(|-2.212|)) = 2(0.0136) = 0.0272. \]

Since \( P < \alpha = 0.05 \), reject \( H_0 \) and conclude that taking vitamin C reduces the incidence rate of colds compared to a placebo.